Howard University Math Department

1. Give a brief (two or three sentences each) answer for the following:

   A. What is wrong with this statement: "People in Chicago are freezing to death this winter. What happened to global warming?"

   B. What are some of the major sources of greenhouse gas emissions causing climate change? How can we reduce their emissions?

   Solution:

   A. Global warming means *average* temperatures are on the rise. Sometimes it can be warmer or colder but on average earth is getting warmer each year. Indeed, a result of global warming is the disruption of the polar vertex bringing arctic air to places like Chicago.

   B. Transportation (cars, planes, etc.,), power plants, agriculture, buildings and factories are some of the major sources. By driving less or driving electric or hybrid cars, using solar and wind energy, reducing meat consumption, making buildings and factories energy efficient we can reduce emissions from these sources.

2. For the following sequences, find the formula for the n-th term and the value of the seventh term using the formulae for the arithmetic sequence or geometric sequence depending on what it is: (a) $1, -1, 1, -1, \ldots$ (b) $1.2, 2.4, 3.6, \ldots$

   Solution:

   (a) This is a geometric sequence because each time we are multiplying by the same number -1.

   So the common ratio is $r = -1$ and the first term $a = 1$.

   Using the formula for n-th term we get $a_n = ar^{n-1} = 1(-1)^{n-1} = (-1)^{n-1}$.

   Seventh term has $n = 7$ and we get $a_7 = (-1)^{7-1} = (-1)^6 = 1$.

   (b) This is an arithmetic sequence because each time we are adding the same number 1.2.

   So the common difference is $d = 1.2$ and the first term $a = 1.2$.

   In $mn + b = a_n$ if we put $n = 1, m = 1.2$ we get $1.2 + b = 1.2$ because $a_1 = 1.2$. Solving, we get $b = 0$ and $a_n = 1.2n$. The seventh term is $1.2 \times 7 = 8.4$.

   Alternatively, using the formula for n-th term we get $a_n = a+d(n-1) = 1.2+1.2(n-1) = 1.2n$.

3. On some planet a ball is moving up according to the equation $y(t) = 10t - t^2$ where t is time in seconds and height is in meters. Find its maximum height y and the time at which it will reach that height.

   Solution

   To get the maximum height we need to find the vertex.

   First note that $a = -1, b = 10$. Because a is negative, the parabola will be facing down.

The t value of the vertex is given by $-b/2a = -10/(2 \times (-1)) = 5$ seconds.

Put $t = 5$ in $y = 10t - t^2$ to get the y value of the vertex.

You get $y = 10(5) - 5^2 = 25$ meters.

This is the maximum height because the parabola is facing down (down because a is negative).

4. The following formulae give the populations of two different cities where t is in years. For each equation, decide whether the relationship is exponential, linear, or neither. In the case of exponential function say what is the growth rate, i.e, by what percentage is the population growing or declining each year. In the case of linear function say by what amount the population is increasing or decreasing.

   (a) $P_A = \dfrac{2}{2^t}$ (b) $P_B = 1000 - 20t$

   Solution:

   (a) The formula $P_A = \dfrac{2}{2^t}$ is exponential and it is decreasing by 50 percent per year because it is reduced by half every time.

   (b) The formula $P_B = 1000 - 20t$ is linear and it is decreasing at a constant rate of 20 people per year. The equation is of the form $y = mx + b$.

5. A phone company charges a flat monthly fee of \$40 and also charges 2 cents per text message. Write down the formula for the cost for n text messages. What is the cost of 100 text messages?

   Solution:

   Total charge = Fixed charge + (number of messages)x(per message charge)

   $= 40 + (n) \times (0.02) = 40 + 0.02n$. Notice that 2 cents have to be written as 0.02 \$.

   If $n = 100$, then $A(100) = 40 + 0.02(100) = \$42$.

6. Suppose the sales of an electric car, say, Stella Zoom,is 100 on January 1st 2019 and increases by 20 each week. How many cars they would have sold totally after 10 weeks?

   Solution:

   Letting $a_1 = 100, m = 20$, we get the first term is $m(1) + b = 20 + b = 100$ which gives $b = 80$.

   So the n-th term $a_n = 20n + 80$.

   Letting $n = 10$ we get

   $$10 \text{ } week \text{ } Total = \frac{1}{2}n(a_1 + a_n) = \frac{1}{2}(10)(100 + (20n + 80)) = 5(20(10) + 180) = 1900.$$

7. Write the exponential function equation for each problem.

   a. The area of a lake's surface covered by algae, initially at 1 million square feet, was reduced by 20 percent each year since the passage of anti-pollution laws.

   b. In 1950, the population of a town was 3000 people. After that the town grew at a rate of 10 percent per year.

Solution:

In general, if something increases or decreases by a percentage or multiple of *the amount at THAT time* then it is modeled by an exponential function.

a. $A(t) = 1(1 - 0.2)^t = (0.8)^t$ million square feet.

b. Increases by 10 percent each time means its equation looks like

$$P(t) = P(1 + r)^t = 3000(1.1^t).$$

8. A solar heater is in the shape of a parabolic dish with the stove located at the focus. If the dish is 8 feet in diameter at its edge and 1 foot deep, where is the focus located? i.e, how far above the vertex is the focus?

   Solution:

   The cross section of the dish is a parabola facing up. We can position it so that its vertex is (0,0) and axis of symmetry is the y-axis. Then its equation would be of form $y = ax^2$. Now, diameter is 8 means distance from axis to edge of parabola is 4. Depth is 1 means height of that edge is 1. So we have (4,1) on the parabola which means $1 = a(4^2)$ giving $a = 1/16$, thus giving Focus at $1/(4a) = 1/(1/4) = 4$ foot from vertex.

9. 1000 dollars is invested in a bank account at an interest rate of 10 per cent per year,

   (a) Find the amount in the bank after 5 years if interest is compounded semi-annually (every six months).

   (b) Find the amount in the bank after 5 years if interest is compounded quarterly.

   (You can leave your answer in the following example format: Amount after 5 years if interest compounded annually is $A(5) = 1000(1.1^5)$.)

   Solution:

   We have $r = 0.1$ (divide 10 by 100 to convert from percentage to fraction).

   Also $P = 1000, t = 10$.

   As discussed in class $A(t)$ is amount after t years with compounding n times a year is given by

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$(a) \quad A(t) = 1000\left(1 + \frac{0.1}{2}\right)^{2t} \implies A(5) = 1000(1.05)^{2\times5} = 1000(1.05)^{10} \text{ dollars.}$$

$$(b) \quad A(t) = 1000\left(1 + \frac{0.1}{4}\right)^{4t} \implies A(5) = 1000(1.025)^{4\times5} = 1000(1.025)^{20} \text{ dollars.}$$

10. If a population is growing by 5 percent *continuously*, write an equation for the population after t years. If the population is 10000 after 13.86 years, what was the original population? You can use $e^{0.693} = 2$ (approximately).

Solution:

The population after t years $P(t) = Pe^{0.05t}$, with P being initial population.

It is just as in the case of compound interest with rate of interest replaced by rate of growth.

To find initial population if population is 10000 after 2 years, solve

$$10000 = P(13.86) = Pe^{0.05\times 13.86} = Pe^{0.693}$$

to get $P = 10000/e^{0.693} = 10000/2 = 5000.$

So the population is halved in 13.86 years.