2-4-2019 Notes, Patterns in Environmental math

Quadratic Equations Review, Parabola

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Outline

General form of quadratic equation, its solution

Definition

A quadratic equation in general: $ax^2 + bx + c = 0$.

GENERAL SOLUTION (QUADRATIC FORMULA)

$$\implies x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

So for example if $a = 1, b = -2, c = 2$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)} = 1 \pm i.$$

Proof of Quadratic Formula

We get it using the *Completing the Square* process:

Given $ax^2 + bx + c = 0$, we first write $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$.

Then we take constant to other side: $x^2 + \frac{b}{a}x = -\frac{c}{a}$.

Now we add the square of half of $x-$coefficient to both sides.

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2.$$

$$\Longrightarrow \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{-4ac + b^2}{4a^2}.$$

Taking square root of both sides we can solve for x and get the Quadratic Formula.

The quadratic function

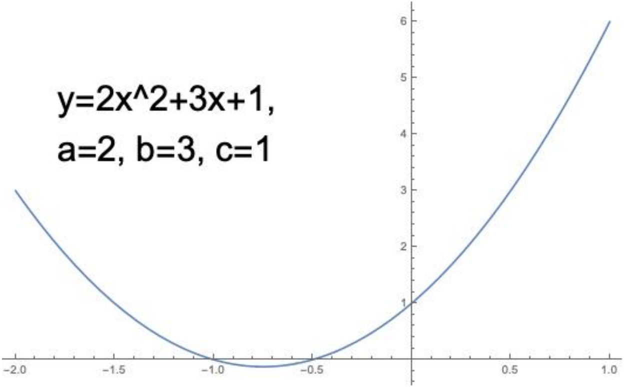
Definition

The general quadratic function: $y = ax^2 + bx + c$.

The graph will be a parabola, facing up if $a > 0$,
and its *vertex* will be on the *axis of symmetry* which will be $x = -b/2a$.
The parabola will cut the $y-$ axis at $y = c$. [Put $x = 0$].
and cut the $x-$axis where $ax^2 + bx + c = 0$ [Put $y = 0$].
You can find these points by factoring $ax^2 + bx + c = 0$
or if that won't work, using the Quadratic Formula.

Example of a Quadratic Function

Graph of a quadratic function will be a parabola.



Problem: Graph $y = x - x^2$.

Draw the graph of the quadratic function $y = x - x^2$.

Is the parabola facing up or down?

Where is its vertex and axis of symmetry?

What are its x and y intercepts?

Solution to problem: Graph $y = x - x^2$.

Draw the graph of the quadratic function $y = x - x^2$.

First write it as $y = -x^2 + x$. So $a = -1, b = 1, c = 0$.
Is the parabola facing up or down? It faces down because $a = -1$ and it is negative.

Where is its vertex and axis of symmetry?
Axis of symmetry is $x = -b/2a = -1/2(-1) = 1/2$.
Vertex is at $x = 1/2, y = (1/2) - (1/2)^2 = 1/4$.

What are its x and y intercepts?
Its x-intercepts are where $y = 0$. That is , $x - x^2 = 0$.
Factoring, we get $x(1 - x) = 0 \implies x = 0, 1$.
0 is also the y-intercept. (To get it put $x = 0$).

What are some places where you will see a parabola?

1. Orbits of comets (mentioned in "Hidden Figures")
2. Headlights of cars, Parabolic microphones
3. Telescopes, Radars
4. Solar thermal power plants

Couple of examples

(Photos from wikipedia)

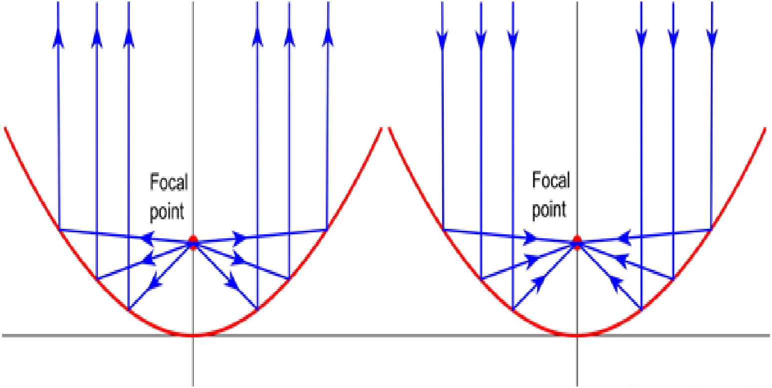
Parabolic trough system

Parabolic dish system

A key property of the parabola

Light (or heat) rays parallel to axis of symmetry will get reflected and **pass through the focus.**

Light (or heat) rays **coming from the focus** will get reflected and come out parallel to axis of symmetry..



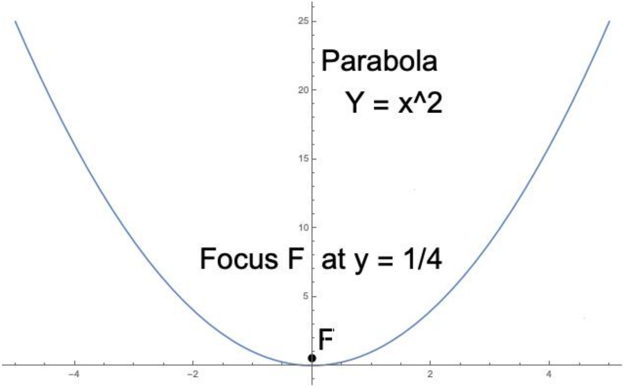
Where is the focus of the parabola?

If the equation is $y = ax^2, \ a > 0$
then the focus is at $y = \dfrac{1}{4a}$.

Note: If equation is $y = ax^2 + bx + c$,
it can be transformed to $y - k = a(x - h)^2$
using completing the square process described earlier.
This will have vertex at (h, k) and focus at $(h, k + \dfrac{1}{4a})$.

Also, if $a < 0$ then parabola faces down. In that case replace $1/4a$ with $-1/4a$ in everything above.

Example of focus of a parabola-1



Example of focus of a parabola-2

This parabola is a bit wider, and its focus is a bit farther from vertex. We have $a = 1/4$ and so focus is at $y = 1/(4a) = 1$.