Fall 2012 Applied Calculus Final Exam
Howard University Math Department

INSTRUCTIONS:
Each problem 20 points
Total time 2 hours
You must provide step by step solutions.

PART I : DO ALL PROBLEMS

1. Find the equation of the tangent line to the graph of $f(x) = \dfrac{x^2}{2x+3}$ at the point where $x = 0$.

2. The total cost of producing q units of a certain commodity is given by
   $C(q) = 0.002q^3 - .05q^2 + 3000$.

   a) What is the total cost to produce 10 units?

   b) Use marginal analysis to estimate the cost of producing the 10th unit.

   Soln: (a) Find $C(10)$. (b) Find $C'(10)$, i.e, find the derivative and then plug in $q = 10$. This estimates cost because $dC = C'(q)dq$ and $dq = 1$ when you are trying to find marginal cost because you are increasing q by just 1.

3. A manufacturer estimates that the demand function for one of its commodities is given by $D(p) = 75 - p^2$ for $0 \le p \le 8$. [i.e, $75 - p^2$ units are sold].

   (a) Express the revenue function in terms of the price p.

   (b) At what price in $0 \le p \le 8$ is the revenue function maximum? Use derivatives to find your answer.

4. For the function $f(x) = 3x^4 - 8x^3 + 6x^2 + 1$:

   (a) Find the critical numbers

   (b) Find the intervals where f is increasing and decreasing.

   (c) Classify the critical numbers as relative maximum and minimum.

5. Find the area between the graph of $f(x) = \sqrt{3x+4}$ and the $x-$axis over the interval $0 \le x \le 4$.

PART II : DO ANY 5 PROBLEMS

6. By using logarithmic differentiation **ONLY**, find the derivative of $g(x) = \dfrac{e^x(x+1)^3}{\sqrt{6-x^2}}$.

Soln: Taking logarithms of both sides, and using properties of logarithms, you get

$$ln(g(x)) = ln(e^x) + ln((x+1)^3) - ln((6-x^2)^{1/2}) = x + 3ln(x+1) - (1/2)ln(6-x^2).$$

Differentiate both sides. On LHS you will get $g'(x)/g(x)$. Then multiply both sides by $g(x)$ to get $g'(x)$. Remember that you need to use chain rule when you differentiate $ln(g(x)$ and $ln(6-x^2)$.

7. Find the absolute maximum and absolute minimum of the function

   $f(x) = x^3 - 12x^2 + 21x$ on the interval $0 \leq x \leq 2$.

   Soln: Find the critical points first. Here the derivative will be well defined everywhere, so you just need to find the points where derivative is zero.

   Using first or second derivative test, determine which of them are minima or maxima.

   Then compare values at those critical points that are minima or maxima and the values at the boundary points 0 and 2 to figure out the absolute maxima and minima.

8. Find the limit of the following functions

   (a) $\lim_{x\to\infty} \dfrac{x^2+1}{x^2-1}$ (b) $\lim_{x\to 1} \dfrac{x^2+x-2}{x^2-1}$

9. For the function $y = xe^{-x}$,

   (a) Find $\dfrac{d^2y}{dx^2}$ $(i.e,\ y''(x))$

   (b) Find the intervals where the graph of y is concave up and concave down.

   (c) Find the inflection points of the graph of y.

   Soln: You need to use product formula to find the first as well as the second derivatives. You get

   $$\begin{aligned} y''(x) &= (y'(x))' = (x(-e^{-x}) + e^{-x})' = -(xe^{-x})' + (e^{-x})' \\ &= -(x(-e^{-x}) + e^{-x}) - e^{-x} = xe^{-x} - 2e^{-x} = (x-2)e^{-x}. \end{aligned}$$

   The inflexion points are where $y''(x) = 0$. But this can happen only when $x - 2 = 0$ or $x = 2$ because e^{-x} is never zero (in fact it is always positive). So 2 is the sole inflection point. Check the sign of $y''(x)$ at one point to the left and one point to the right of zero to see where it is concave up or down.

10. Find the derivatives of the following functions:

    a) $f(x) = (2x^2 - 5x)\sqrt{x-4}$ (b) $f(x) = \dfrac{e^{-x}}{x + \ln x}$.

11. Use implicit differentiation to find dy/dx at $x = 0, y = 2$ given that

$$x^2y - 6y^2 + 5x = -24.$$

12. Find the function y satisfying the differential equation $dy/dx = xe^{x^2}$ and such that $y(0) = 1$.

13. Use a Riemann sum using left endpoints, with 5 rectangles of equal width, to approximate the area under the graph of $f(x) = \frac{1}{x}$ over the $x-$axis between $x = 1$ and $x = 2$. Compare with actual value of area calculated using integral obtained using the fundamental theorem (i.e, the usual definite integral using anti-derivative).

14. Integrate by parts $\int (x+1)(x+2)^5 dx$.

15. Using x skilled workers and y unskilled workers, a manufacturer can produce

    $Q(x, y) = 25xy^2$ units per day. Currently, there are 20 skilled workers and 30 unskilled workers on the job.

    a) How many units are currently being produced?

    b) By how much will the daily production level change if 1 more skilled worker is added to the current work force?

    c) By how much will the daily production level change if 1 more unskilled worker is added to the current work force?

    d) By how much will the daily production level change if 1 more skilled worker and 1 more unskilled worker are added to the current work force?