Wikipedia assignment: history of some algebra problems?

Examples of fields

Example : Show that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\}$ is a field.

This is an example of a *quadratic field extension* of $\mathbb{Q}$.
Recall that it is isomorphic to $\mathbb{Q}[x]/(x^2 - 2)$. More on extensions in 5.3.

To show existence of inverses, use "rationalization of denominator."

Problems 9 and 10, 5.1 *Show that if F is a finite field of characteristic p then $\phi : F \mapsto F$ given by $\phi(x) = x^p$ is an automorphism.*

This map is called the *Frobenius* endomorphism.
Note that ϕ is an injective homomorphism for any F with $char(F) = p$.

Example of a field where ϕ is NOT an automorphism:

Let $\mathbb{F}_p = \mathbb{Z}/p\mathbb{Z}$, the finite field with p elements, p being prime.
Then $\mathbb{F}_p$ is a field of characteristic p.
The field of rational functions with $\mathbb{F}_p$ coefficients, denoted as $\mathbb{F}_p(x)$, is also of characteristic p.
This is the field of quotients of the integral domain $\mathbb{F}_p[x]$.
But if you take $x \in \mathbb{F}_p[x]$ then $x = u^p$ is not possible for any $u \in \mathbb{F}_p[x]$.

HW7: 4.7: 2,3,4 ; 5.1: 3,4,5