

Field of quotients, Field extensions

Theorem: Given an integral domain D we can construct a field F such that $D \subset F$ and F is the *Field of Quotients* of D .

Example: Construction of \mathbb{Q} from \mathbb{Z} .

Field of Quotients: Equivalence classes of pairs of elements (a, b) in D with $b \neq 0$ given by the relation $(a, b) \sim (c, d)$ iff $ad = bc$.

Addition and multiplication of equivalence classes defined as :

$$[a, b][c, d] = [ac, bd] \quad [a, b] + [c, d] = [ad + bc, bd].$$

Show that the addition is associative.

HW7:

4.7: 2,3,4

5.1: 3,4,5