CALCULATING MORTGAGE PAYMENTS

Let initial amount (of loan, mortgage, etc.,) be P.

Let the periodic payments be each equal to R.

Each payment first includes the interest on the *remaining balance.*

The *rest of the payment goes towards paying off P.*

Let I_1 be the interest included in the first payment.

$I_1 = Pi$ where i is the interest rate.

Let P_1 be the amount in the first payment that goes towards paying off P.

Then $P_1 = R - I_1 = R - Pi$.

Let I_2 be the interest included in the second payment.

This is calculated on the remaining balance, which is $P - P_1$, because the amount P1 has already been paid off.

So $I_2 = (P - P_1)i = Pi - P_1 i = I_1 - P_1 i$.

Now P_2, the amount remaining in the second payment after paying off I_2 is :

$P_2 = R - I_2 = R - (I_1 - P_1 i) = R - I_1 + P_1 i$.

But $R - I_1 = P_1$ from above! So $P_2 = P_1 + P_1 i = P_1(1 + i)$.

Next we look at the third payment:

The interest in the third payment is $I_3 = (P - P_1 - P_2)i = Pi - P_1 i - P_2 i = I_2 - P_2 i$.

Now P_3, the amount remaining in the third payment after paying off I_3 is :

$P_3 = R - I_3 = R - (I_2 - P_2 i) = R - I_2 + P_2 i$.

But $R - I_2 = P_2$ from above! So $P_3 = P_2 + P_2 i = P_2(1 + i)$.

So you can see a pattern emerging already.

It can be proved, in the same manner as above, $I_4 = I_3 - P_3 i,\ P_4 = P_3(1 + i)$, and so on.

If we continue like that, when we come to the *last payment,* after paying P_n and the interest I_n, nothing should remain!

Moreover, $I_n = P_n i$ because P_n is all that remains in P.

So we must have $R = P_n + P_n i = P_n(1 + i)$ which means $P_n = \frac{R}{1+i}$.

Then from the last but one step, $P_n = P_{n-1}(1 + i)$ we get $P_{n-1} = \frac{P_n}{1+i} = \frac{\left(\frac{R}{1+i}\right)}{1+i} = \frac{R}{(1+i)^2}$.

Continuing like this, we will get $P_{n-2} = \frac{R}{(1+i)^3}$ and so on until we get to $P_1 = \frac{R}{(1+i)^n}$.

Now we have $P = P_1 + P_2 + P_3 + ... + P_n = \frac{R}{(1+i)^n} + \frac{R}{(1+i)^{n-1}} + ... + \frac{R}{(1+i)^2} + \frac{R}{(1+i)}$

This is a geometric sequence with $a = \frac{R}{(1+i)}$ and $r = \frac{1}{1+i}$.

The sum is $P = \sum_{k=1}^{n} \frac{R}{(1+i)^k} = \frac{R}{(1+i)} \frac{(1-\frac{1}{(1+i)^n})}{1-\frac{1}{1+i}}$.

Simplifying this, and solving for R we will get $R = \frac{Pi}{(1-\frac{1}{(1+i)^n})}$.