

Howard University Math Department

4/18/2012

College Algebra II Quiz 8

Spring 2012

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

Each problem 10 points.

Challenge problem is extra credit 10 points.

1. Find the number of five letter words that can be made using A,B,C,D,E,F,I,L,M,N if:
 - (a) No letter is repeated
 - (b) repetition is allowed

Solution:

(a) This is the number of permutations of 10 things taken 5 at a time.

Put $n = 10, r = 5$ in $nPr = \frac{n!}{(n-r)!}$ to get $\frac{10!}{5!} = 10 \times 9 \times 8 \times 7 \times 6 = 30240$.

(b) Because repetition is allowed, each letter can be chosen in 10 ways. So the answer is $10^5 = 100,000$.

2. In a 6/49 lottery, 6 numbers are selected out of 49. You win the jackpot if your ticket has all six numbers, regardless of order. Find the probability of winning the jackpot.

Solution: Total number of ways to select 6 out of 49 regardless of order is given by ${}_{49}C_6 = \frac{49!}{(49-6)!6!} = \frac{49!}{43!6!} = 13983816$.

Probability of winning = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = 1/13983816 = 7.15 \times 10^{-8}$.

3. Find the probability of getting 2 Kings and 3 Aces in a card hand of five. [There are 4 Kings and 4 Aces in a deck of 52 cards].

Solution: Many of you did this problem by adding the probability of getting two kings and the probability of getting three aces. But that works only when you are dealing with two mutually exclusive events. Here we have a single event involving the picking of five cards in a hand. This problem would be hard to do by breaking down into independent events and multiplying probabilities. You can, if you wish, try to see how many ways you can pick the first card and so on, but it would be hard.

Probability of 2 Kings and 3 Aces =

$$= \frac{\text{Number of ways to select 2 Kings and 3 Aces}}{\text{Number of ways to select 5 cards}}$$

$$\begin{aligned}
&= \frac{4C_2 \times 4C_3}{52C_5} = \frac{4!}{2!2!} \times \frac{4!}{3!1!} / \frac{52!}{5!47!} \\
&= (6 \times 4) / \frac{52 \times 51 \times 50 \times 49 \times 48}{5!} = \frac{24}{259860} = 1/108290 = 0.0000092
\end{aligned}$$

4. Find the number of ways of selecting a committee with one Chairperson, one Vice-Chair and 4 members from a class of 20 students.

Solution:

The Chair and the Vice-Chair can be selected in $20 \times 19 = 380$ ways. The remaining four can be selected in

$$18C_4 = \frac{18!}{14!4!} = \frac{18 \times 17 \times 16 \times 15}{4 \times 3 \times 2 \times 1} = 3060 \text{ ways.}$$

So total number of ways to select committee equals the product of the two which is 1162800.

We multiply instead of add because we are doing both at the same time – picking the Chair and Vice-Chair and then the 4 members.

5. (Challenge) Find the number of ways of arranging 10 science books on a shelf if the math book and the physics book always have to be together. How many ways can you arrange them if the math book and physics book are never together?

Solution: Since the two books are together, considering them as one book, we get the number of ways of arranging the 8 remaining books plus the "one book" as $9!$. Now the math and physics books can be arranged in two ways—either math on left and physics on right or vice versa. So totally we get $2 \times 9! = 725760$ ways to arrange all the books.

To count the number of ways to arrange them if they are never together, we simply subtract the above answer (number of ways to arrange them when they are together) from the total number of ways to arrange all 10 books, which is $10! = 3628800$. We get the answer as $3628800 - 725760 = 2903040$.