Howard University Math Department

4/9/2012 College Algebra II Quiz 7 Spring 2012

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.
Challenge problem is extra credit 10 points.

1. (a) Find the amount accumulated in an investment account if \$1000 is deposited each year for 30 years at 5 percent annual return rate.

   (b) (bonus 5 points) How much money was actually deposited (without including interest)?

   Solution:

   (a) The amount is given by $A_f = R((1+i)^n - 1)/i$ with

   $R = 1000, i = .05, n = 30.$

   We get $A_f = 1000(1.05^{30} - 1)/(.05) = 66,438.85$

   (b) The actual amount deposited was 30,000 (every year \$ 1000 for 30 years).

2. If a car costs \$ 20,000 and is financed by a loan at 3 percent interest for 4 years, find the monthly payment. Use the formula $R = \dfrac{Pi}{1 - \left(\frac{1}{1+i}\right)^n}$.

   Solution:

   We have $P = 20,000, i = .03/12 = 0.0025, n = 4 \times 12 = 48.$

   We get

   $$R = \frac{20000(0.0025)}{1 - \left(\frac{1}{1.0025}\right)^{48}} = 442.69$$

3. Write the term with A^5 in the expansion $(A-3)^8$.

   Solution: It is okay to use Pascal's triangle for this.

   Using binomial theorem:

   Here $n = 8, k = 5$. The expansion will look like $(A-3)^8 = A^8 + 8A^7(-3) + ...$

   The desired term will be of the form

   $$\binom{8}{5} A^5(-3)^3 = \frac{8!}{3! \times 5!} A^5(-27) = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(5 \times 4 \times 3 \times 2 \times 1)} A^5(-27)$$

   $$= \frac{8 \times 7 \times 6}{3 \times 2 \times 1} A^5(-27) = 56A^5(-27) = -1512A^5.$$

4. Find the number of possible outfits a girl can create from 12 shirts, 4 pairs of jeans and 6 pairs of shoes.

   Solution:

   12 times 4 times 6 = 288.

5. (Challenge) Notice that if you subtract alternate terms in Pascal triangle you get zero: $1-2+1=0, 1-3+3-1=0, 1-4+6-4+1=0$ and so on. Prove this is true for all n as follows: Expand $(1-1)^n$ using the binomial theorem and show that

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + ... + (-1)^n \binom{n}{n} = 0$$

   Solution: Let $x = 1, y = -1$ in the binomial formula

$$(x+y)^n = \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + ... + \binom{n}{n-1} x y^{n-1} + \binom{n}{n} y^n$$

   You get

$$(1-1)^n = \binom{n}{0} 1^n + \binom{n}{1} 1^{n-1}(-1) + ... + \binom{n}{n-1} 1(-1)^{n-1} + \binom{n}{n} (-1)^n$$

   The left hand side is 0 because $0^n = 0$ for all n.

$$\text{The right side is } \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + ... + (-1)^n \binom{n}{n}$$