Howard University Math Department

3/21/2012 **College Algebra II Quiz 6** **Spring 2012**

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.
Challenge problem is extra credit 10 points.

1. Find the coordinates of the foci, vertices, asymptotes and graph the hyperbola given by the equation $x^2 - 4y^2 = 16$.

   Solution:

   Dividing all by 16 we get $\frac{x^2}{16} - \frac{y^2}{4} = 1$.

   Since x^2 has the positive coefficient, this hyperbola has axis along the $x-$axis.

   Also we have $a^2 = 16, b^2 = 4, c^2 = a^2 + b^2 = 20$.

   So $a = 4, b = 2, c = \sqrt{20}$.

   So the vertices are (4,0) and (-4,0). The foci are at $(\sqrt{20}, 0)$ and $(-\sqrt{20}, 0)$.

   The asymptotes are given by $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$, namely $y = \frac{x}{2}$ and $y = -\frac{x}{2}$.

2. Find the coordinates of the foci, vertex, axis of symmetry and graph the parabola given by the equation $(x - 1)^2 = 4y - 8$. You must label the vertex, focus and the axis in the graph.

   Solution:

   Writing this as $(x - 1)^2 = 4(y - 2)$ we get the vertex as $(h, k) = (1, 2)$.

   Since this is of the form $x^2 = 4py$ it is vertical and the focus is given by $p = 1$.

   So the parabola will face up, with vertex at (1,2), focus at (1,3) [focus is 1 unit above the vertex].

   You can also get this by (1,2)+(0,1) = (1,3). Here (0,1) is the old focus when vertex is at (0,0).

   The axis of symmetry is the vertical line through (1,2), namely $x = 1$.

3. Write the formula for the general term a_n of the sequence 1,8,27,64,125,....Your answer must be a function of n.

   Solution:

   $a_n = n^3$. Check by putting $n = 1, 2, 3, 4, 5, ...$

4. Find the sum

$$\sum_{k=1}^{5} 2^n.$$

Solution:

$$\sum_{k=1}^{5} 2^n = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 = 2 + 4 + 8 + 16 + 32 = 62.$$

5. (Challenge) Malia borrows $10,000 from her uncle. She agrees to repay him $ 200 per month plus 0.5 percent interest on the previous balance. So after she pays $200 in the first month the balance left is $A_1 = (10000 - 200) + 10000(0.005) = 9850$. After the second payment of $ 200 the balance left is $A_2 = (9850 - 200) + 9850(0.005) = 9699.25$ and so on. Write a recursive formula for the balance A_n after the $n-$th payment. [Your answer should be in terms of A_{n-1}].

   Solution: This is problem 74 in the book. The recursive formula is obtained as follows:

$$A_n = A_{n-1} - 200 + A_{n-1}(0.005) = A_{n-1} + A_{n-1}(0.005) - 200 = A_{n-1}(1.005) - 200.$$