Howard University Math Department

3/2/2012 **College Algebra II Quiz 5** **Spring 2012**

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.
Challenge problem is extra credit 10 points.

1. Solve the two equations using elimination:

$$3x^2 + y = 5 \quad (1)$$
$$2x^2 - y = 15 \quad (2)$$

Solution:

(1)+(2) gives $5x^2 = 20$ which means $x^2 = 4$ or $x = 2$ or -2. Then we get $y = -7$ upon plugging in $x = 2$ or $x = -2$. So there are two solutions: (2,-7) and (-2,-7).

2. Solve the system of inequalities. You must indicate the boundaries of your solution region and mention the vertices.

$$y + x^2 \leq 0 \quad (3)$$
$$y + x \geq -2 \quad (4)$$

Solution:

The first inequality is the parabola facing down with vertex at the origin (0,0).

The points satisfying ≤ 0 are the points below and inside the parabola.

The second inequality is a line passing through (0,-2) and (-2,0).

The points satisfying ≥ -2 are those above the line.

This cuts the parabola at the points where $y = -x^2 = -x - 2$. [Get $y = -x - 2$ from (4) and put that into (3)]. Solving $x^2 - x - 2 = 0$ we get $x = 2, x = -1$. Plugging in these values of x we get $y = -4, y = -1$. So the vertices are (2,-4) and (-1,-1).

So the desired solution region is a bounded region enclosed by the parabola above and the line below, between the points (0,0),(-1,-1) and (2,-4).

3. A parabolic mirror with a vertical axis of symmetry has focus at a distance of 2 units above its vertex. Write the equation of the parabola and find the diameter of the opening of the parabola at a height of 8 units above its vertex.

Solution:

The equation of the parabola is of the form $x^2 = 4py$. Since $p = 2$, the equation becomes $x^2 = 8y$.

Putting $y = 8$, we get $x^2 = 64$ which means $x = 8$. Diiameter of the opening is twice the $x-$value on one side. So the diameter is 16 units.

4. Find the coordinates of the foci and the eccentricity of the ellipse with equation

$$\frac{x^2}{100} + y^2 = 1.$$

Solution:

Looking at the denominators, we get $a^2 = 100, b^2 = 1$.

So major axis is of length $a = 10$, minor axis is of length $b = 1$.

$c^2 = a^2 - b^2 = 99$. So $c = \pm\sqrt{99}$.

Since the major axis is along $x-$axis, the foci are at $(\sqrt{99}, 0), (-\sqrt{99}, 0)$.

Eccentricity is $\sqrt{99}/10$.

5. (Challenge) An ellipse shaped hall 100 feet in length is to be designed as a whispering gallery. Jim, standing at one focus, can hear his friend who is standing at the other focus 50 feet away. How high is the ceiling at the center?

Solution: Based on the information, $a = 50, c = 25$. So $b = \sqrt{a^2 - c^2} = \sqrt{50^2 - 25^2} = 25\sqrt{3} = 43.3$ feet.