Howard University Math Department

2/17/2012 **College Algebra II Quiz 4** **Spring 2012**

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.
Challenge problem is extra credit 10 points.

1. Multiply the two matrices to find AB, if possible:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 & 2 \\ 1 & -1 \\ 3 & 1 \end{bmatrix}$$

Solution:

A is a 2×3 matrix while B is a 3×2 matrix.

Since A has the same number of columns as B has rows, we can multiply them.

Their product AB will be a 2×2 matrix.

If $AB = [a_{ij}]$ then the entry in first row, first column, namely a_{11} will be given by multiplying first row of A with first column of B.

We get $a_{11} = (1 \times 0) + (2 \times 1) + (-1 \times 3) = 0 + 2 - 3 = -1$.

Similarly we find a_{12}, a_{22}, a_{21} and write the matrix AB :

We get

$$AB = \begin{bmatrix} -1 & -1 \\ -2 & 8 \end{bmatrix}.$$

2. Find the inverse by row reducing, if the inverse exists:

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution:

The augmented matrix is

$$A = \begin{bmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Reducing this as we did in class, we get

$$A = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{-3}{2} & 1 & \frac{-3}{2} \\ 0 & 0 & 1 & \frac{-1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

From this we get that

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{-3}{2} & 1 & \frac{-3}{2} \\ \frac{-1}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

3. A clothing company sells jeans, shirts and dresses. They have stores in DC, NY and LA. The number sold in each location on a given day is given in the following table. The price charged for the three items are unknown, but each item has the same price at all three locations. On this day the DC store makes \$ 1000, the NY store makes \$ 1500 and the LA store makes \$ 2000.

   (a) Write an equation in 3 variables (corresponding to the price on each item) representing the above data.

   (b) Write the equation in (a) as a matrix equation involving a 3×3 matrix and two 3×1 matrices (vectors).

   YOU DO NOT NEED TO SOLVE THE EQUATIONS!

DC	NY	LA
15	10	10
20	15	10
15	25	20

Solution:

Let x be the price on jeans, y the price on shirts and z the price on dresses.

The three equations are:

$$15x + 20y + 15z = 1000 \quad (1)$$
$$10x + 15y + 25z = 1500 \quad (2)$$
$$10x + 10y + 20z = 2000 \quad (3)$$

The matrix equation is $AX = B$ where

$$A = \begin{bmatrix} 15 & 20 & 15 \\ 10 & 15 & 25 \\ 10 & 10 & 20 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1000 \\ 1500 \\ 2000 \end{bmatrix}$$

4. Solve the following system using Cramer's rule:

$$10x + 7y = 15 \tag{4}$$
$$8x + 9y = 10 \tag{5}$$

Solution:

We get the matrix equation as

$$\begin{bmatrix} 10 & 7 \\ 8 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 10 \end{bmatrix}$$

Let $det(A) = \begin{vmatrix} 10 & 7 \\ 8 & 9 \end{vmatrix}$. Then $det(A) = (10 \times 9) - (7 \times 8) = 34$.

By Cramer's rule we get

$$x = \frac{\begin{vmatrix} 15 & 7 \\ 10 & 9 \end{vmatrix}}{det(A)} \; ; \; y = \frac{\begin{vmatrix} 10 & 15 \\ 8 & 10 \end{vmatrix}}{det(A)}$$

Calculating all the determinants, we get the solution as $x = \frac{65}{34}, y = \frac{-20}{34}$

5. (Challenge) Say whether there is a unique solution or not to the system in problem 3 without actually solving it, using only the 3×3 matrix you get from the table. Will the inverse of that matrix exist?

   Solution: The determinant of the 3×3 matrix A equals 1000. Since determinant is non-zero the system will have a unique solution AND the inverse will also exist.