Howard University Math Department

1/27/2012 **College Algebra II Quiz 2** **Spring 2012**

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.
Challenge problem is extra credit 10 points.

1. The population of a bacteria colony grows according to the formula $P(t) = 100e^{0.1t}$ where t is measured in hours.

   (a) Find the time it takes for the population to double.

   (b) When would the population equal 400?

   Solution:

   1a. Doubling time $= ln2/r = ln2/(0.1) = 6.93$ hours.

   1b. Initial population is 100 (put $t = 0$). According to (a), population would become 200 in 6.93 hours. Then it will double to 400 in another 6.93 hours! So a total of 13.86 hours.

   You can also say: $P(t) = 400 = 100e^{0.1t}$ means $4 = e^{0.1t}$ after canceling 100.

   Taking ln of both sides, we get $ln4 = 0.1t$ which gives $t = ln4/(0.1) = 13.86$ hours.

2. Looking only at the coefficients, say what rational numbers can possibly be the roots of $2x^3 + x^2 + 1$. Check each one to see if it is a root by plugging it into the polynomial.

   Solution:

   The numerator of the roots must divide 1 (the constant term) and the denominator must divide 2 (the leading coefficient). So the possible numerators are 1 and -1. The possible denominators are 1, -1, 2 and -2. Combining, we get the possible roots to be $1, -1, 1/2$ and $-1/2$. Note that $\frac{1}{-2} = \frac{-1}{2}$.

   Checking $x = 1:\ 2(1)^3 + 1^2 + 1 = 4$. Since $f(1) \neq 0$, 1 is not a root.

   $x = -1:\ 2(-1)^3 + (-1)^2 + 1 = 0$. Since $f(-1) = 0$, -1 is a root.

   $x = 1/2:\ 2(1/2)^3 + (1/2)^2 + 1 = 3/2$. Since $f(1/2) \neq 0$, 1 is not a root.

   $x = -1/2 : 2(-1/2)^3 + (-1/2)^2 + 1 = 1$. Since $f(-1/2) \neq 0$, 1 is not a root.

3. Find all the real roots of $x^3 + x^2 + x + 1$.

   Solution:

Using the rational roots theorem (i.e, looking at the constant term and leading term) we get that the only possible *rational* roots are 1 and -1.

Putting $x = 1$ and $x = -1$ respectively, we see that 1 is not a root but -1 is a root.

Now we divide by $x - -1 = x + 1$. We get the quotient as $x^2 + 1$.

$x^2 + 1$ has no real number solutions. You can see this using quadratic formula : $\sqrt{b^2 - 4ac} = \sqrt{-4}$ is not a real number.

You can also just say: $x^2 + 1 = 0$ means $x^2 = -1$ which is impossible.

So only -1 is a root.

4. Graph $f(x) = \frac{x^2+1}{x^2-1}$. You must label the intercepts and the horizontal and vertical asymptotes.

   Solution:

   The $x-$intercepts are obtained by putting $y = f(x) = 0$.

   The rational function will equal zero for a particular value only if numerator is zero AND the denominator is not zero at that value.

   Setting the numerator equal to zero we get $x^2 + 1 = 0$ which is impossible. So there are no $x-$intercepts.

   The $y-$intercept is attained at $x = 0$. When $x = 0$ you get $y = -1$.

   The horizontal azymptotes are obtained by dividing by highest power and looking at what happens as x goes to ∞. [Both negative and positive ∞].

   Dividing above and below by x^2 we get $\frac{1+\frac{1}{x^2}}{1-\frac{1}{x^2}}$.

   This goes to 1 as x goes to $+\infty$ or $-\infty$. So the line $y = 1$ is a horizontal asymptote on both ends of the $x-$axis.

   The vertical asymptotes are obtained by setting denominator equal to 0.

   $x^2 - 1 = 0$ means $x^2 = 1$ which gives $x = 1$ or -1.

   The function will go to either ∞ and $-\infty$ near 1 and -1.

   By looking at values for x near 1 and -1 we see that it will go to $+\infty$ to the left of -1 and $-\infty$ to the right of -1 and left of 1. It will go to $+\infty$ again to the right of 1.

   Just for this problem, it is okay to say that by looking at the graph, there is no $x-$intercept because outside of (-1,1) the graph is above $y = 1$, the horizontal asymptote. Inside of (-1,1), graph is below -1.

5. (Challenge) The product of 3 consecutive natural numbers equals 720. Find the 3 numbers. You must use algebra.

   Solution: Let x be the smallest of the three. Then the other two are $x = 1$ and $x = 2$.

   We get $x(x + 1)(x + 2) = 720$.

Simplifying, we get $x^3 + 3x^2 + 2x - 720 = 0$.

This has one change of sign, so only one positive root. Since natural numbers are positive, it means there is only one natural number that will satisfy it.

By the rational roots theorem, the root must divide 720.

Possibilities are 1,2,3,4,5,6,8,9,10,12 etc.,

But as we look at this list we see that 8,9,10 are consecutive numbers and their product is 720.

So 8,9,10 is the answer.