Howard University Math Department

1/18/2012 **College Algebra II Quiz 1** **Spring 2012**

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 8 points. Challenge problem is extra credit.
Any points you get in excess of 40 is extra credit.

1. Evaluate each expression. Show how you arrive at your solution.

   (a) $log_2 16$.

   (b) e^{ln10}

   Solution:

   1a. $16 = 2^4$. So $log_2 16 = 4$.

   1b. Answer is 10. e^x and lnx are inverses of each other, so they "cancel " each other out.

2. Convert the following exponential equation to logarithmic equation and solve it:

   $3^{x+1} = 27$.

   Solution:

   You get $log_3 27 = x+1$. But because $3^3 = 27$, we get $log_3 27 = 3$. Thus $log_3 27 = x+1 = 3$ which means $x = 2$.

3. Find the domain of $log(x-5)$ and graph it.

   Solution:

   The domain is all the values for which $x - 5 > 0$ or $x > 5$.

   This can be represented in interval form as $(5, \infty)$.

   The graph will be same as that of $logx$ except shifted 5 units to the right. It will pass through (6,0) and will go to $-\infty$ as you approach the vertical line $x = 5$ which is the asymptote.

4. Simplify and write as a single logarithm: $ln800 + ln15 - 3ln10$.

   Solution:

   First note that $3ln10 = ln10^3 = ln1000$.

   Next, using laws of logarithms, we get $ln800 + ln15 - ln1000 = ln\frac{800\times15}{1000} = ln12$.

   Note that $\frac{ln12000}{ln1000}$ is not the same as $ln(\frac{12000}{1000})$.

5. An amount of P dollars is invested at 10 percent and interest is compounded annually. Show that after t years it will grow to $P(1.1^t)$ dollars. Find out how long it will take it to equal $2P$. In other words, how long would it take to double? Note that you *don't need to know* what P is! Doubling time is same no matter what amount you start with, for a given value of r.

   Solution: $r = 0.1, n = 1$, so $A(t) = P(1 + \frac{r}{n})^{nt} = P(1 + 0.1)^t = P(1.1^t)$.

   To get to $2P$, we have $2P = P(1.1^t)$ which gives $2 = 1.1^t$

   Taking logarithms of both sides, $ln2 = tln1.1$ which means $t = ln2/ln1.1 = 7.27$ years or about 7 years, 3 months.

6. (Challenge) The decibel level of sound is measured using the following formula:

$$d = 10log(I/I_0)$$

   Here d is the number of decibels of a sound of intensity I and I_0 is the intensity of the lowest sound human ears can detect. If the sound at a stadium I_s is measured as 100 decibels, and that of a car I_c is measured at 80 decibels, how much bigger is I_s compared to I_c ? [Remember: logx means $log_{10}x$.]

   Solution: We have, using the formula, the following equations:

$$100 = 10log(I_s/I_0)$$
$$80 = 10log(I_c/I_0).$$

   Subtracting, we get $20 = 10[log(I_s/I_0) - log(I_c/I_0)]$.

   Canceling the 10 on both sides and using the property $logA - logB = log(A/B)$ we get

$$2 = log(\frac{\frac{I_s}{I_0}}{\frac{I_c}{I_0}}) = log(I_s/I_c).$$

   Converting this to an exponential equation, we get $10^2 = 100 = I_s/I_c$.

   This means that the sound at the stadium was 100 times that of the car.