3/10/2011 Spring 2011, Calculus III Quiz 5 Sitaraman

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
ANSWERS WITHOUT EXPLANATION WILL ONLY GET 40 percent
Time Limit 30 minutes ; Total 50 points
Please read the questions carefully before answering
It is recommended that you try those problems you are most comfortable with, first.
Attempt as many as you can; Anything over 50 is extra credit.

1. (10 points) Say whether each statement is true or false. In each case, explain why it is true/false or give counter-example. If true it is NOT enough to give just one example.
(a) Level surfaces of a function of three variables $f(x, y, z)$ are always obtained by graphing $f(x, y, z) = 0$.
(b) $\underset{(x,y)\to(0,0)}{Lim} \frac{x-y}{x+y} = 1$.

Soln:
1a) FALSE. Level surfaces of a function of three variables $f(x, y, z)$ are obtained by graphing $f(x, y, z) = k$ and k need not always be 0.
1b) FALSE. This limit does not exist. If you approach $(0, 0)$ along the $y-$axis you get
$$\underset{(x,y)\to(0,0)}{Lim} \frac{x-y}{x+y} = \underset{(0,y)\to(0,0)}{Lim} \frac{-y}{y} = -1.$$
If you approach $(0, 0)$ along the $x-$axis you get
$$\underset{(x,y)\to(0,0)}{Lim} \frac{x-y}{x+y} = \underset{(x,0)\to(0,0)}{Lim} \frac{x}{x} = 1.$$

2(a). (10 points) Describe the domain of $f(x, y, z) = ln(x^2 + y^2 + z^2)$.
2(b). (10 points) Describe the largest region in which the function $\frac{x-y}{x+y}$ is continuous.

Soln:
2a) Need $x^2+y^2+z^2 > 0$ in order for $ln(x^2+y^2+z^2)$ to be defined. This set is the set of all points in $\mathbf{R}^3$ other than (0,0,0) because $x^2+y^2+z^2 > 0$ for all other values of (x, y, z). So the domain is all points in 3-space other than (0,0,0).

2b)Both numerator and denominator are polynomials. So this function is continuous wherever the denominator is not zero. So wherever $x+y \neq 0$ the function is continuous. Hence the answer is: the set of all points of the plane not on the line $x+y=0$.

3a. (10 points) Given that $f(x,y) = xye^x$ find $f_x, f_y, f_{xx}, f_{xy}, f_{yy}$.
3b.(10 points) Given that $w = f(x,y,z)$ and $w^4+x^4+y^4+z^4 = 0$ find $\frac{\partial w}{\partial x}$ using implicit differentiation. Explain why you can write down $\frac{\partial w}{\partial y}, \frac{\partial w}{\partial z}$ using the answer for $\frac{\partial w}{\partial x}$ without actually doing implicit differentiation again.

Soln:

3(a): $f_x = y(xe^x)_x = y(e^x + xe^x) = ye^x + yxe^x = ye^x(x+1)$.

$f_y = xe^x(y)_y = xe^x$.

$f_{xx} = (f_x)_x = (ye^x + yxe^x)_x = ye^x + y(xe^x)_x = ye^x + ye^x + yxe^x = ye^x(x+2)$.

$f_{yy} = (f_y)_y = 0$ because there is no y in f_y.

$f_{xy} = (f_x)_y = (ye^x(x+1))_y = e^x(x+1)$.

Note that $f_{yx} = (f_y)_x = (xe^x)_x = e^x + xe^x = e^x(x+1) = f_{xy}$.

Solution for (b):

Differentiating implicitly we get $4w^3\frac{\partial w}{\partial x} + 4x^3 = 0$.

Solving, we get $\frac{\partial w}{\partial x} = -4x^3/(4w^3) = -x^3/w^3$.

Since the equation is symmetrical in x, y, z, by looking at this answer we can say that

$\frac{\partial w}{\partial y} = -y^3/w^3$, $\frac{\partial w}{\partial z} = -z^3/w^3$.

4. (Challenge, 15 points) The legs of a right triangle are 3 cm and 4 cm, with a maximum error of 0.05 cm in each measurement. Use differentials to approximate the maximum possible error in the value of the hypotenuse and the area of the rectangle.

Soln:

Let x and y represent the lengths of the legs. Let $A(x,y) = (xy)/2$ be the area and $H(x,y) = \sqrt{x^2+y^2}$ be the hypotenuse.

Then we have $dA = A_x dx + A_y dy = \frac{y}{2}dx + \frac{x}{2}dy = (3/2)(0.05) + (4/2)(0.05) = 0.175$. This is approximately the maximum possible error in the calculated value of the area.

Similarly $dH = H_x dx + H_y dy = 2x\frac{(1/2)}{\sqrt{x^2+y^2}}dx + 2y\frac{(1/2)}{(1/2)\sqrt{x^2+y^2}}dy = \frac{3(0.05)+4(0.05)}{5} = 0.07$ is approiximately the maximum error in the hypotenuse.