Spring 05 Test 2 Solutions Calculus III 158-01 Sitaraman 2/28/05

1. (30 points) State whether the following are true or false and explain why, in each case, either using proof (if it is true) and using example (if false):

a) If two vectors lie in different planes, their dot product cannot be zero.

b) The scalar moment of a force about a point P increases as you move the point Q where the force is applied away from the point P, as long as the direction of the force remains the same.

c) The work done by a force is maximum when the force is applied in a direction parallel to the motion of the body.

Solution: (a) False. The vector (0,1,0) lies in the x-y plane. But the vector (0,0,1) which is in the x-z plane (or y-z plane) has dot-product with (0,1,0) equal to zero.

(b) False. The scalar moment is given by the product of the magnitude of the force with the distance PQ and the sine of the angle between PQ and the direction of the force. So even though the direction of the force is unchanged, and hence the angle and the sine of that angle, we are not given that the magnitude of the force is the same. Though the length PQ increases, it is possible that the magnitude of the force decreases and the scalar moment does not increase.

(c) True. Work done depends on the magnitude of the force along the direction of the motion, and this is maximum when the force is in the same direction as the motion.

2. (30 points) A boat is going east on a river. The current is pushing it with a force of 100 N in the easterly direction. If now a steady wind were to blow with a force of 50 N in the North-eastern direction (making 45 degrees with the river), what would be the resulting acceleration and direction of the boat? (Assume the boat weighs 1000 Kg, and use Force = (mass)(acceleration)).

Soln: With E as positive x-direction and N as positive y direction, we can write the force of the current as $100\mathbf{i}$ and the force of the wind as $50cos(\pi/4)\mathbf{i}+50sin(\pi/4)\mathbf{j}$. resultant force is $\mathbf{F} = (100+25\sqrt{2})\mathbf{i}+25\sqrt{2}\mathbf{j}$. Its magnitude is $\sqrt{(100+25\sqrt{2})^2+(25\sqrt{2})^2} = 139.897N$. it makes an angle of $(100 + 25\sqrt{2})/(139.897) = \pi/(12.2959)$ *rad* or 14.639 degrees with the x-axis. ($cos\theta = (\mathbf{F}.\mathbf{i})/(||\mathbf{F}||)$.) Acceleration of the boat is given by (Force)/(mass) which is $139.897/1000 = .139897$ m/sec/sec.

3. (From www.epinions.com/auto-review-7F2-1A400ECA-3A1C5CA2-prod2)

Every damn performance car ad touts its horsepower figures. Horsepower has a better PR agent than N'Sync. What you WILL need is another, somewhat lesser-advertised quantity that is a direct factor in propelling your ride from a dead stop to your desired velocity with expedience. This quantity is the Father of horsepower. TORQUE is what you need! Torque matters! Horsepower is just a wannabe follower.The definition of torque is the rotational equivalent of force. Force is further defined as mass times acceleration, or, how much is pushing on something. Torque is the potential to produce a twisting, or rotational force. The rotational force exerted by the engine on the crankshaft is what they call the Torque of the car. The more force that is fed to the wheel, the greater the rate of acceleration.Torque is a vector value and can be calculated by the cross-product of Force and Distance from center. From where it is produced at the rear of the engine's crankshaft, torque is transmitted through the transmission and through the rest of the driveline to the wheels. The weight of the car and the size of the wheels determines how much force is needed to get the wheels turning and the car to move. The more force that is fed to the wheel, the greater the rate of acceleration.

Okay, now the math question: If a car requires 3000 lb-feet of torque, and the piston pushes the crankshaft at a distance of half a foot from its centre perpendicular to the rod joining the piston to the center of the crankshaft, how much force should it apply?

Soln: Magnitude of Torque is the magnitude of cross product of force vector with PQ vector where P is the point about which force is applied (here, center of crankshaft) and Q is the point at which force is applied (here Q is at a distance of 0.5 foot from P). This magnitude is given by $Fdsin\theta$ where F is magnitude of force, d is length of PQ, θ is angle between PQ vector and force vector. So d is 0.5, $\theta = 90$ degrees. Thus we want $T = 3000 = F(.5)(sin90) = F/2$. Thus the force must equal 6000 lb.

4. Find the parametric equation of the intersection of the two planes 2x+y-z=0 and x-y+z=0.

Soln: Adding the two equations, we get $3x = 0$ and hence $x = 0$. Letting $z = t$, we get $y = t$. So the equation of the line of intersection is $\mathbf{r}(t) = t\mathbf{j} + t\mathbf{k}$.

5. Find the graph of the line segment given by the vector equation $\mathbf{r}(t) = (1 - t)(\mathbf{i} - \mathbf{j}) + t(\mathbf{i} + \mathbf{j})$.

Soln: Graph will be the line segment from (1,-1) to (1,1) with t increasing as we go up the segment.