Spring 04 Test 1 solutions Calculus III 158-01 Sitaraman 2/2/05

1. (30 points) Assume that an ellipse has center (0,0) and major axis along the x-axis. Graph it, showing the co-ordinates (for instance, one vertex is (a,0)) of the vertices and the foci. Write its equation in terms of x,y, a and b. Solve for y (assume y is positive, or in the first quadrant). Find the slope of the tangent to the ellipse at the point whose x-coordinate is c (i.e, this point is directly above the focus).

Soln: Equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Solving for y (in the first quadrant only) we get $y = (b)\sqrt{1 - \frac{x^2}{a^2}}$. Slope at (x, y) is $\frac{dy}{dx} = (b)(\frac{-2x}{a^2})/(\sqrt{1 - \frac{x^2}{a^2}})$. The slope at $x = c$ is thn given by $(-2bc)/((a^2)(\sqrt{1 - \frac{c^2}{a^2}})$. But $a^2 - c^2 = b^2$, so $\sqrt{1 - \frac{c^2}{a^2}} = b/a$. Plugging this back into the slope, we get slope at $x = c$ is $(-2bc)/(ab) = -2c/a$.

2. (30 points) Change the co-ordinate axes to find the equation of the hyperbola $xy = 1$ in standard format (in the new co-ordinates). Draw a graph of the hyperbola, showing clearly the new co-ordinate axes, and locate the vertices and the foci on these axes.

3. Write the polar equation of the parabola with directrix 3 units to the right of the focus. Use this equation to find the point on the parabola that makes an angle of 45 degrees with the focus (which is at the pole). What happens when the angle is 180 degrees? Why?

4. Draw a sketch of the cube with one corner at (0,0,0) and the opposite corner at (1,1,1) with the x, y, z axes forming three edges. Write down the vectors representing all the four diagonals that pass through the center of the cube, always starting at a corner on the x-y plane. For instance, the diagonal starting from $\mathbf{a} = (0, 0, 0)$ and ending at $\mathbf{b} = (1, 1, 1)$ is represented by the vector $\mathbf{b} - \mathbf{a} = (1, 1, 1)$. What are their lengths?