Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (a) (6 points) Show that the following is an equivalence relation on the rational numbers, written such that the numerator and denominator have no common factors :

$$xRy \quad \iff x, y \text{ have the same denominator.}$$

NOTE:

Assume that the denominator of 0 is 1 (so $0 = 0/1$).

Example for no common factors : $2/4$ is written as $1/2$ after cancelling out the 2.

(b) (9 points) Find the equivalence class of 1, 1.4 and 1/3.

Easy to check that this is an equivalence relation.

The denominator of 1 is 1 and its equivalence class is all integers. For example, if $x = a/b, y = c/d, z = e/f$ then xRy means $b = d$ and yRz means $d = f$ and the two together give $b = f$ which means xRz. This shows it is transitive. The other two are even easier.

The denominator of $1.4 = 14/10 = 7/5$ is 5 and its equivalence class is all rational numbers of the form $m/5$ where m is an integer not divisible by 5.

The denominator of $1/3$ is 3 and its equivalence class is all rational numbers of the form $m/3$ where m is an integer not divisible by 3.

2. (15 points) Given the sequence $s_n = 5, 11, 17, 23, ..., 6n - 1, ...$ answer the following:

(a) Is it increasing, decreasing, nonincreasing or nondecreasing? (could be more than one).

(b) Find $\sum_{n=1}^{4} s_n$

(c) Find the formula for the k-th term of the subsequence $s_2, s_4, s_6, ...$ (with even indices only) considered as a new sequence $a_1, a_2, a_3, ...$

Solution:

a) It is increasing and nondecreasing.

b) $5 + 11 + 17 + 23 = 56$.

c) We have $n = 2k$ and plugging it into $s_n = 6n - 1$ we get $a_k = s_{2k} = 12k - 1$.

Check: $a_1 = s_2 = 11 = 12 - 1, a_2 = s_4 = 23 = 24 - 1, ...$

3. (Extra credit) (8 points) Find the partition of the rational numbers created by the equivalence relation of problem 1.

   (7 points) What kind of sequence is 5, 11, 17, 23, ... ? what would be the formula for the n-th term if it started with 11? (as it is $s_n = 6n - 1$).

   The given equivalence relation partitions $\mathbb{Q}$ into numbers of the form x/n where n is a fixed positive integer and x is any integer that has no common factor with n.

   It is an arithmetic sequence with common difference 6. If it started with 11 then you replace n with $n + 1$ because the 2nd term becomes the first, 3rd term becomes the second, and so on. So you get $s_n = 6(n + 1) - 1 = 6n + 5$.