

12/1 Final Exam Version 3, Discrete Structures, Fall 2025
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You must show all your work to get full credit. No cellphone allowed.
Total points 200. (To be converted to out of 100 for final grade).

- (30 points) A survey of 1000 college students asked about their coffee choices. 350 students reported they do not drink coffee. 450 students reported that they put cream in their coffee. 500 students reported that they put sugar in their coffee. 375 students reported that they put both cream and sugar in their coffee. How many students drink their coffee black (with no cream, no sugar)?
- (20 points) Check whether the following argument is logically correct. Does the last sentence follow from the first?
A: If a natural number n divides the product ab of two natural numbers, then it divides one of them.
B: n does not divide a
C: Therefore n does not divide ab .
(20 points) Is the statement A true by itself? Prove or give counterexample.
- (30 points) Show that the relation fRg iff $f(1) = g(1)$ is an equivalence relation on the set of linear functions $F(x) = mx + b$ on the real number set. Find the equivalence class of $f(x) = 1$ and that of $g(x) = x + 1$.
- (30 points) Using regular induction show that for all natural numbers n ,
$$\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1) \cdot (2n+1)} = \frac{n}{2n+1}$$
- (40 points) (a) How many ways can six friends A, B, C, D, E, F be seated at a table?
(b) How many ways to seat them if A and B are always seated together, in the order AB?
(c) How many ways to seat them if A and B are never seated together?
(d) How many ways to distribute 10 identical bottles of water to 5 people??
- (30 points) Check if it is possible to have an Eulerian cycle on a graph with five vertices a, b, c, d, e and edges ab, bc, cd, da, ac, bd, be, ec. You must use graph theory. Write down the adjacency matrix for this graph.
- (15 points extra credit) Prove using induction that the number of subsets of a set of n elements is 2^n .