

Howard University Math Department

PLEASE PROVIDE STEP BY STEP SOLUTIONS.

TOTAL 100 POINTS. TIME 50 MINUTES.

1. (20 points) Find GCD of 198 AND 121 using (a) unique factorization (b) Euclidean algorithm and then (c) find x, y such that $121x + 198y = \gcd(121, 198)$.
2. (20 points) Can there be infinitely many primes of the following forms: $n^3 - 1, n^3 + 1, n^4 - 1, n^4 + 1, n^2 + n + 2$?
3. (20 points) Show that GCD of three integers a, b, c is the minimum positive of all linear combinations $ax + by + cz$.
4. (20 points) (a) Solve the equation $ax \equiv 1 \pmod{5}$ for $a = 1, 2, 3, 4$ using any method, with $x \in \{1, 2, 3, 4\}$. (b) Why are the solutions unique? (c) Show that the set $\{0, 1, 2, 3, 4\}$ is a field under addition and multiplication mod 5.
5. (20 points) Explain how you parametrize rational points on the unit circle using rational points on the y -axis. Find a Pythagorean triple by mapping $P = (0, 5/6)$ to the unit circle under the parametrization map.