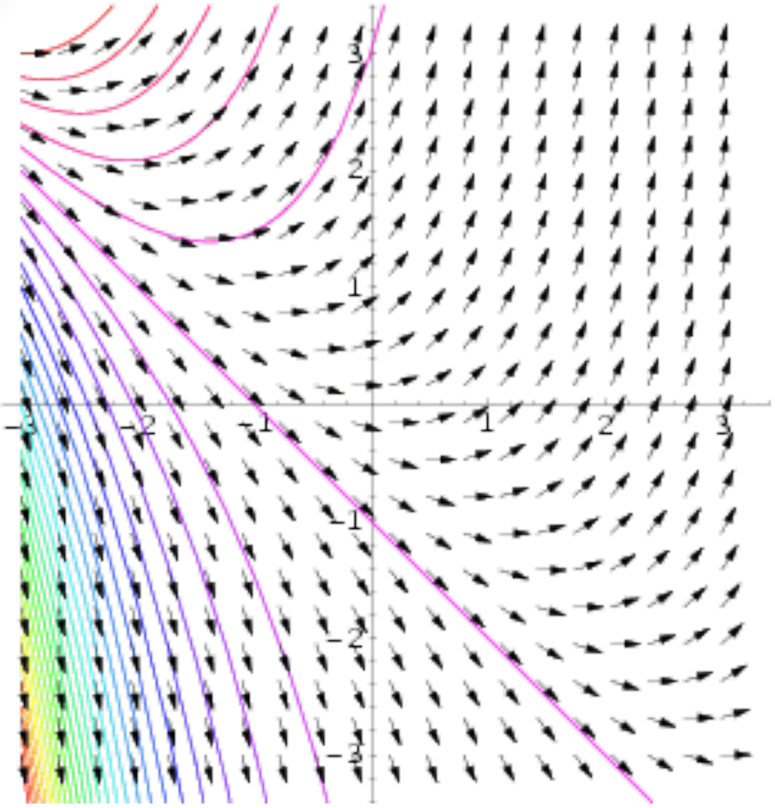
WHY DO WE NEED INTEGRATION?

INTEGRATION

1.FIGURE OUT FUNCTION FROM ITS RATE OF CHANGE

2. ADD VALUES OF A FUNCTION, ESPECIALLY INFINITELY MANY

SLOPE FIELD SHOWING RATE OF CHANGE



- Each little arrow represents direction and size (usually by its length) of slope of a function at that point
- This is actually slope field of a bunch of curves, each the graph of a function

Examples

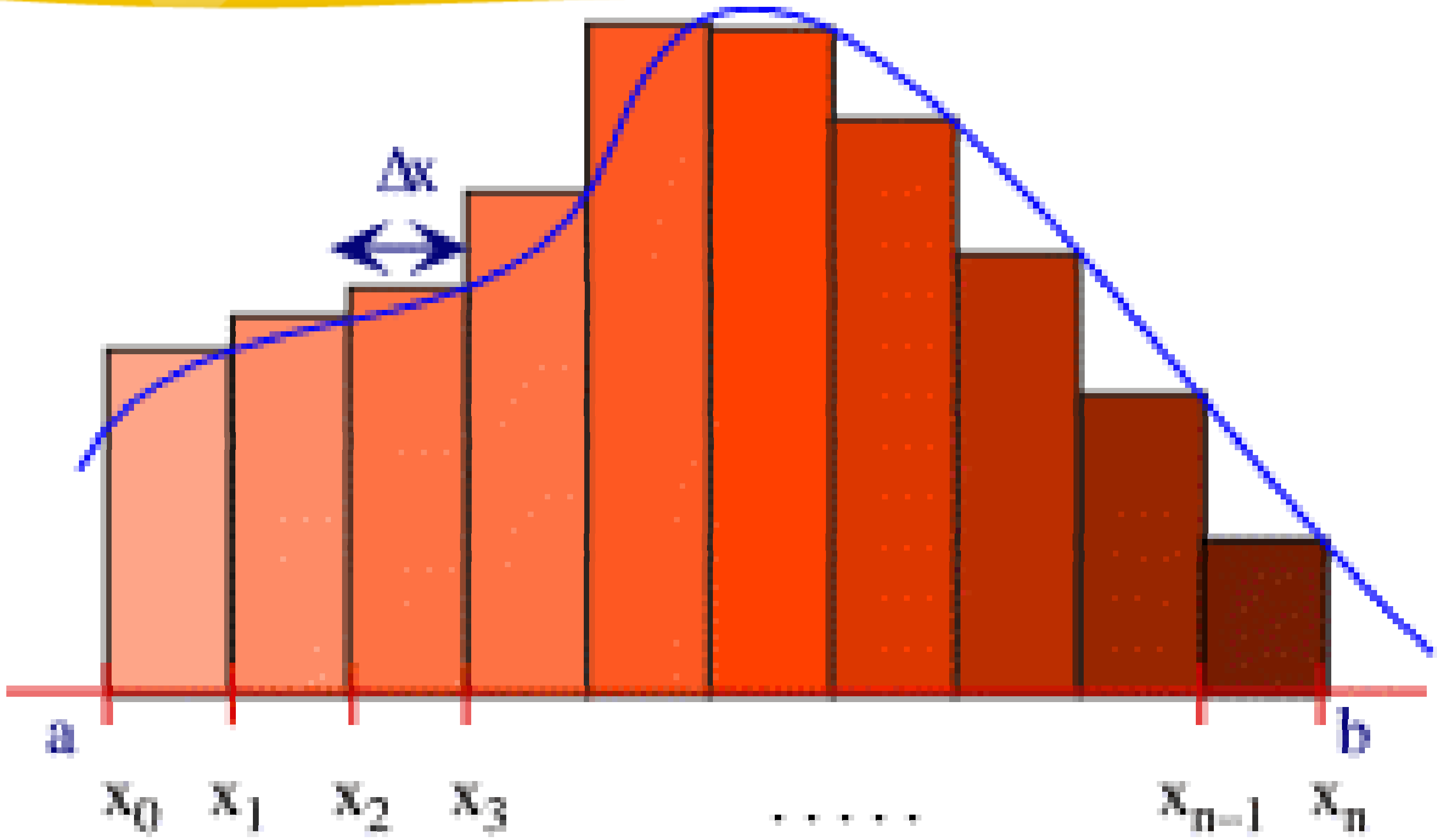
- P' (t) = Growth of population in one year (*very approximately*)
- Knowing P' (t) for several years in a row, could try to figure out P(t)
- Similarly for revenue function, etc
- Velocity at a point
- V(t) = S' (t) where S(t) is position at time t
- Could try to figure out S(t) using V(t)

INTEGRAL OF $Y = X^2$

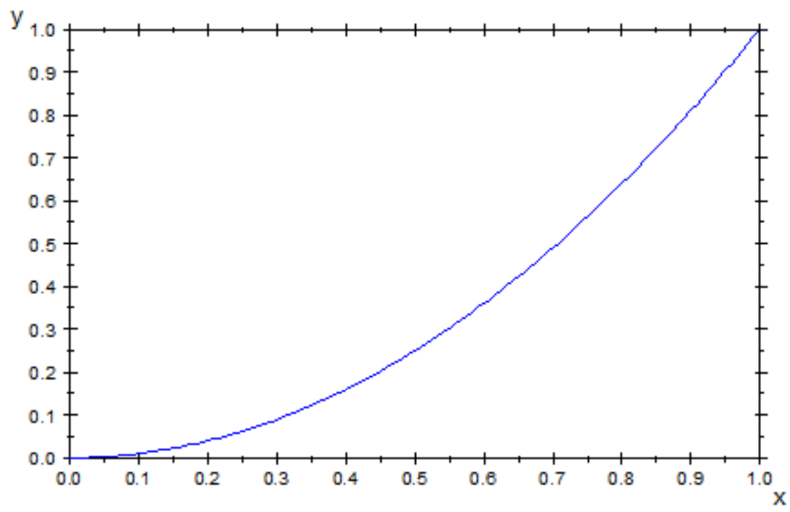
- LET US SAY THE SLOPE OF A CURVE F(X) IS GIVEN BY X^2 FOR ALL VALUES OF A FUNCTION
- SO $F'(X) = X^2$
- THEN JUST BY GUESSING WE SEE THAT $F(X) = X^3/3$. (DERIVATIVE OF $X^3/3 = 3X^2/3$

  $= X^2$)

ADDING VALUES OF A FUNCTION

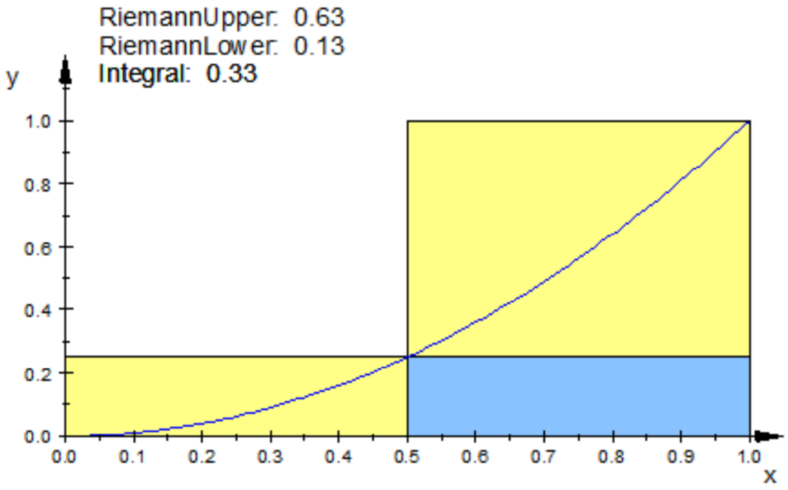


PARABOLA $Y = X^2$



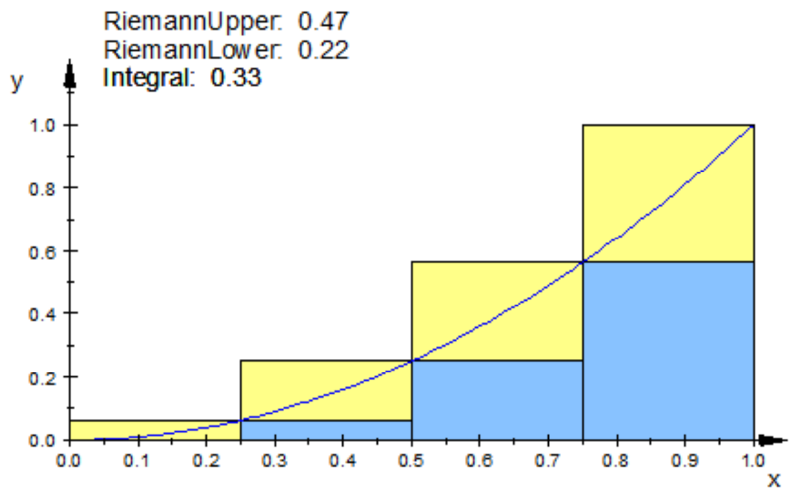
- THIS SHOWS GRAPH FOR X =0 TO X=1
- INFINITE NUMBER OF X-VALUES FROM 0 TO 1
- WANT TO ADD Y-VALUES (OR FIND AREA UNDER CURVE)

IDEA: CHOP AREA INTO BLOCKS



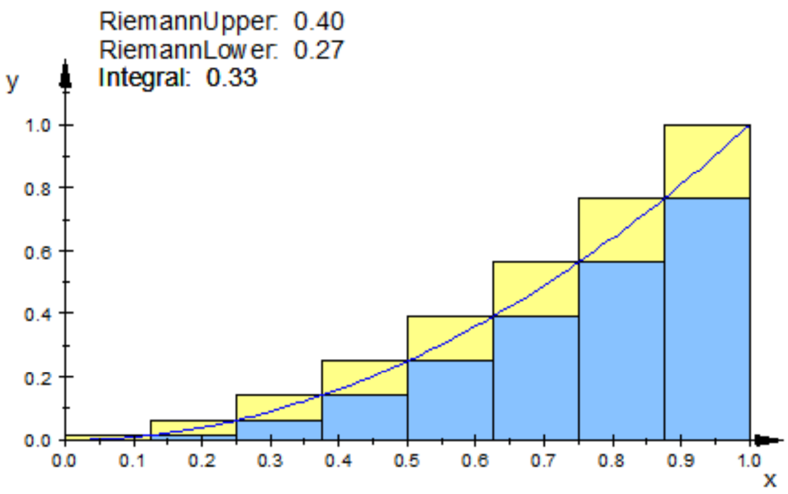
- AREA UNDER CURVE APPROXIMATED BY AREA OF BLOCKS
- AREA OF UPPER BLOCKS
- = (.5)(.25) +(.5)1 = .625
- AREA OF LOWER BLOCKS
- = .25(.5) = .125
- ACTUAL AREA = .33

CHOPPING INTO MORE BLOCKS



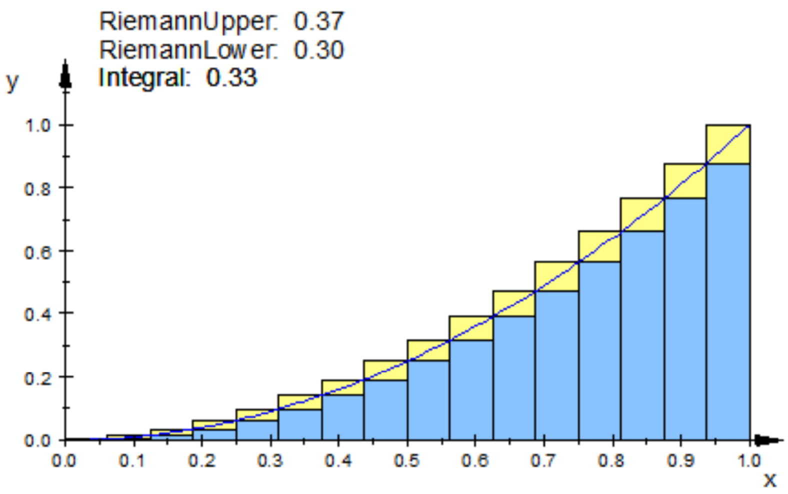
- NOW WE CHOP AREA INTO FOUR BLOCKS, EACH OF LENGTH .25
- TOTAL AREA OF UPPER
- (.25)(.0625+.25+.5625+1)
- =0.46875 = 0.47 APPROX
- TOTAL AREA OF LOWER
- (.25)(.0625+.25+.5625)
- =0.21875 = 0.22 APPROX
- GETTING CLOSER!

CHOPPING EVEN FINER



- AREA OF UPPER BLOCKS (EACH OF LENGTH 0.125) = 0.40 APPROX
- AREA OF LOWER BLOCKS = 0.27 APPROX
- ACTUAL AREA = 0.33 -- EVEN CLOSER!

AND EVEN FINER…!



- AREA OF UPPER = 0.37 APPROX
- AREA OF LOWER
- =0.30 APPROX
- ACTUAL AREA = 0.33 -- ALMOST THERE!

MAKING IT EXACT

- AS YOU CHOP INTO MORE AND MORE BLOCKS THE AREA OF BLOCKS GETS CLOSER AND CLOSER TO ACTUAL AREA

MAKING IT EXACT

- HOW DO YOU MAKE IT EXACT?

ANSWER: TAKE LIMITS!

- AREA = LIMIT OF SUM OF AREA OF BLOCKS, AS N GOES TO INFINITY
- N BEING NUMBER OF BLOCKS

NOW FOR THE PUNCHLINE

- WHAT IS THE RELATION BETWEEN AREA UNDER $Y = X^2$
- OBTAINED USING BLOCKS
- AND INTEGRAL OF $X^2 = X^3/3$ (BECAUSE $(X^3/3)' = X^2$)

GRAND CONCLUSION (END OF MOVIE)

- X=0 TO X=1 WE HAVE (1^3/3)-(0^3/3) = 1/3 = 0.33 (APPROX)
- BY USING BLOCKS WE GOT AREA = 0.33 (APPROX)
- SO THEY ARE THE SAME!
- THIS IS CALLED FUNDAMENTAL THEOREM OF CALCULUS