8/27/2018 Fall 2018, Calculus I Quiz 1 Solutions Sitaraman

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1. The function $f(x)$ is defined by

$$f(x) = \begin{cases} 2 - x & x \leq 1 \\ \dfrac{1}{x} & x > 1 \end{cases}$$

(a) (9 points) Find $f(-1), f(0), f(1), f(2)$ and graph the function.

(b) (6 points) Find its domain.

Solution: Note: Piecewise functions are defined differently on different intervals. This function is defined by $2 - x$ in the interval $x \leq 1$ and by $1/x$ when $x > 1$. In other words, the graph of the function is the graph of $2 - x$ to the left of 1 and its graph is that of $1/x$ to the right of 1.

1a. To find $f(-1)$, we note that $-1 < 1$ and therefore we need to use $f(x) = 2 - x$. So we get $f(-1) = 2 - (-1) = 3$.

To find $f(0)$, we note that $0 < 1$ and therefore we need to use $f(x) = 2 - x$. So we get $f(0) = 2 - 0 = 2$.

To find $f(1)$, we note that $1 = 1$ and therefore it is in $x \leq 1$ and we need to use $f(x) = 2 - x$. So we get $f(1) = 2 - (1) = 1$.

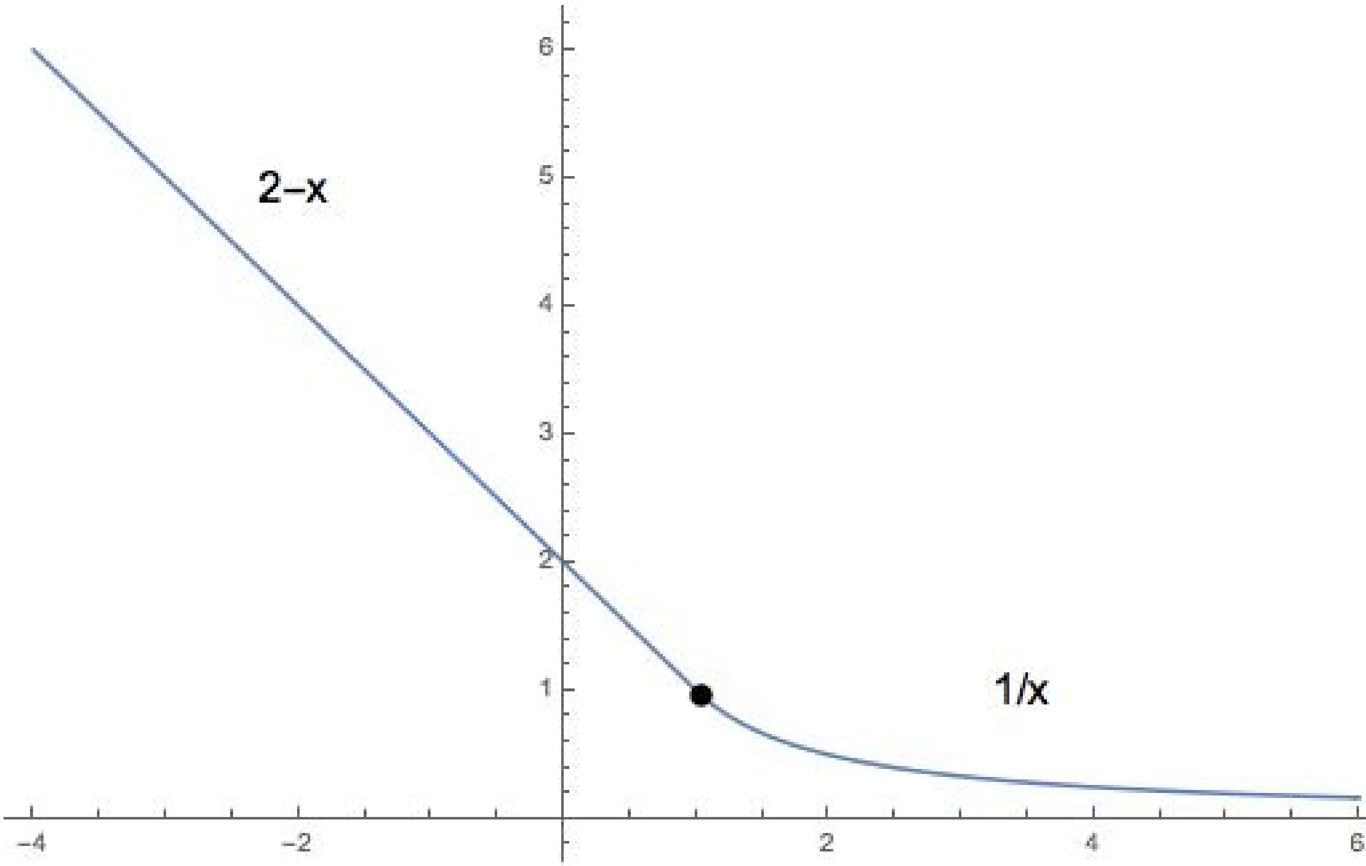
To find $f(2)$, we note that $2 > 1$ and therefore we need to use $f(x) = 1/x$. So we get $f(2) = 1/2$.

The graph is given below. Note that at 1 the value will be 1 whether you use the first definition (i.e, $2 - x$) or the second (i.e, $1/x$).

The value at 1 is 1 and that is indicated by a solid dot in the graph below.

1b. The domain of this function is actually all real numbers, $(-\infty, \infty)$ in interval notation, because for each value of x you will get a valid output. Even though you have $1/x$ in the definition, that is only for $x > 1$, and when $x = 0$ we actually get $f(0) = 2$ as we saw in (a). So we don't need to eliminate 0 from the domain just because $1/x$ is there.

Also remember that a piecewise defined function is *just one function* and we find the domain for the whole function. We don't find the domain for each piece.



2. Given $f(x) = x^2 + 2x + 3$, do the following:

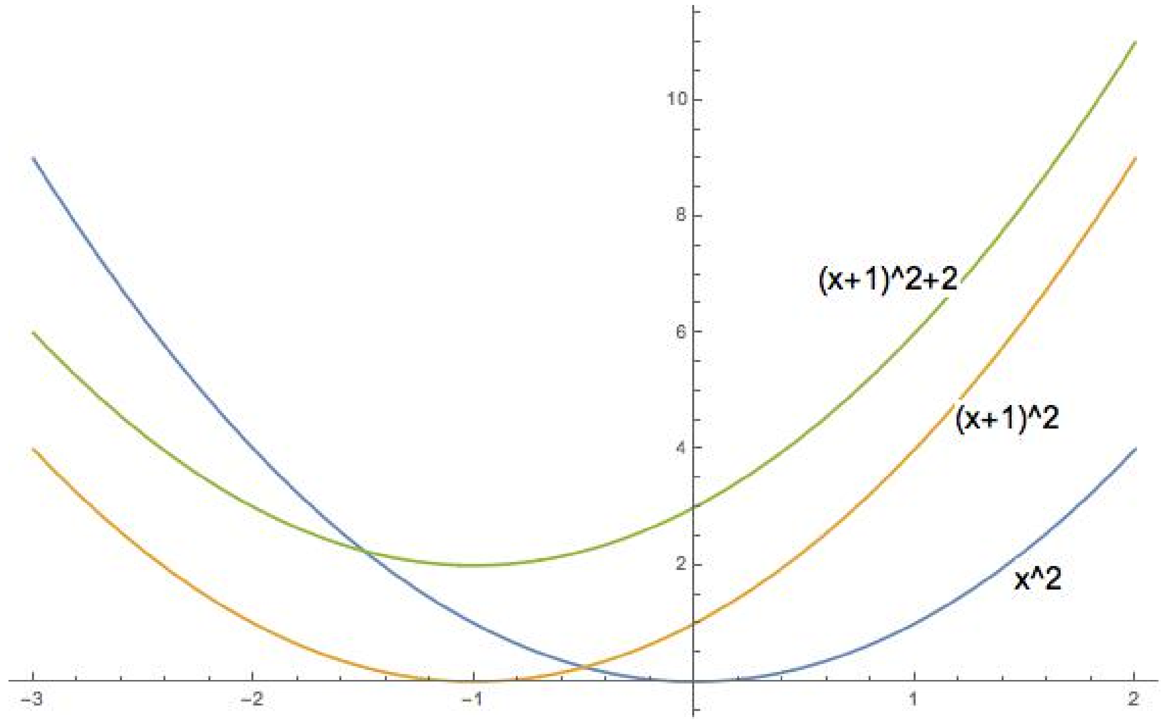
   (a) (5 points) Show that $f(x)$ is of form $(x+a)^2 + b$ by completing the square. What are a and b ?

   (b) (10 points) Draw the graph of $f(x)$ in two steps: By moving the graph of x^2 to get the graph of $(x+a)^2$ and then move the graph of $(x+a)^2$ to get the graph of $(x+a)^2 + b$.

Solution:

2a. To complete the square (provided the quadratic expression starts with x^2 and not $2x^2$ or $3x^2$ or something else times x^2, which is the case here) we divide the $x-$coefficient by 2 and the result would be our a. Here we get $a = 2/2 = 1$. So in this we have $x^2+2x+3 = (x+1)^2 - (1^2) + 3$. We subtracted 1^2 because $(x+1)^2 = x^2+2x+1^2$ and here we only have x^2+2x. So finally we get $x^2+2x+3 = (x+1)^2+3-1 = (x+1)^2+2$. So we have $a = 1, b = 2$.

2b. This is also in the picture below. To get the graph of $(x+1)^2$ from x^2 we shift graph of x^2 to the left by 1 and then to get the graph of $(x+1)^2 + 2$ we move the graph of $(x+1)^2$ up by 2.



10
8
6
4
2
-3
-2
-1
1
2
(x+1)^2+2
(x+1)^2
x^2