

8/24/2018 class notes

FIRST QUIZ ON MONDAY , ON 1.1 TO 1.3

Section 1.2: We will not be spending much time on this. Please read this by yourself. It is quite easy to read.

Section 1.3. New functions from old.

Part I: Transformation of functions:

We can create a new function from a given $f(x)$ in many ways.

*Remember to do this for **each value of x !!***

1. TRANSLATION ALONG x axis:

When x is changed to $x+c$, the graph is shifted left or right depending on whether c is positive or negative. For example, graph of $(x+1)^2$ is obtained by shifting graph of x^2 to the left by 1 unit. For $(x-1)^2$ you shift it right by 1 unit.

2. TRANSLATION ALONG y-axis:

Change $y = f(x)$: For this you get a new function by changing the y -value itself.

Your new function will look like $f(x)+c$ and its graph can be obtained from that of $f(x)$ by moving it up or down depending on whether c is positive or negative.

3. REFLECTION ABOUT y-axis

When x is changed to $-x$, the graph of original function is flipped about y -axis

4. REFLECTION ABOUT x-axis

When y or $f(x)$ is changed to $-y$ or $-f(x)$, the graph of original function is flipped about x -axis.

5. STRETCHING AND SHRINKING:

When x or y is multiplied by a **fixed number** c you either stretch or shrink it horizontally depending on whether $c > 1$ or $c < 1$.

WARNING: $-x^2$ IS NOT SAME AS $(-x)^2$
ALSO, $(A+B)^2$ IS NOT A^2+B^2

Below I have included graphs of x^2 , $2x^2$ and $x^2/2$



When multiplying x or y by a constant factor c it is enough to try a few values to get an idea of how the graph is transformed. No need to memorize what happens in each case.

Can you graph (Problem 1.3.15) x^2-4x+5 starting from graph of x^2 and x , and also graph (Problem 1.3.20) $|x|-2$ starting from graph of $|x|$? What about $|x+2|$?

Answers at bottom of this document

Part II. Combinations of functions.

You can make new functions also by combining two or more functions.

You can add or subtract, multiply or divide the values of two functions to get new functions. You need to do this for *every value of x , though*.

Only thing to watch out for is that $f(x)/g(x)$ is not defined at x if $g(x) = 0$.

There is also **composition of two functions**.

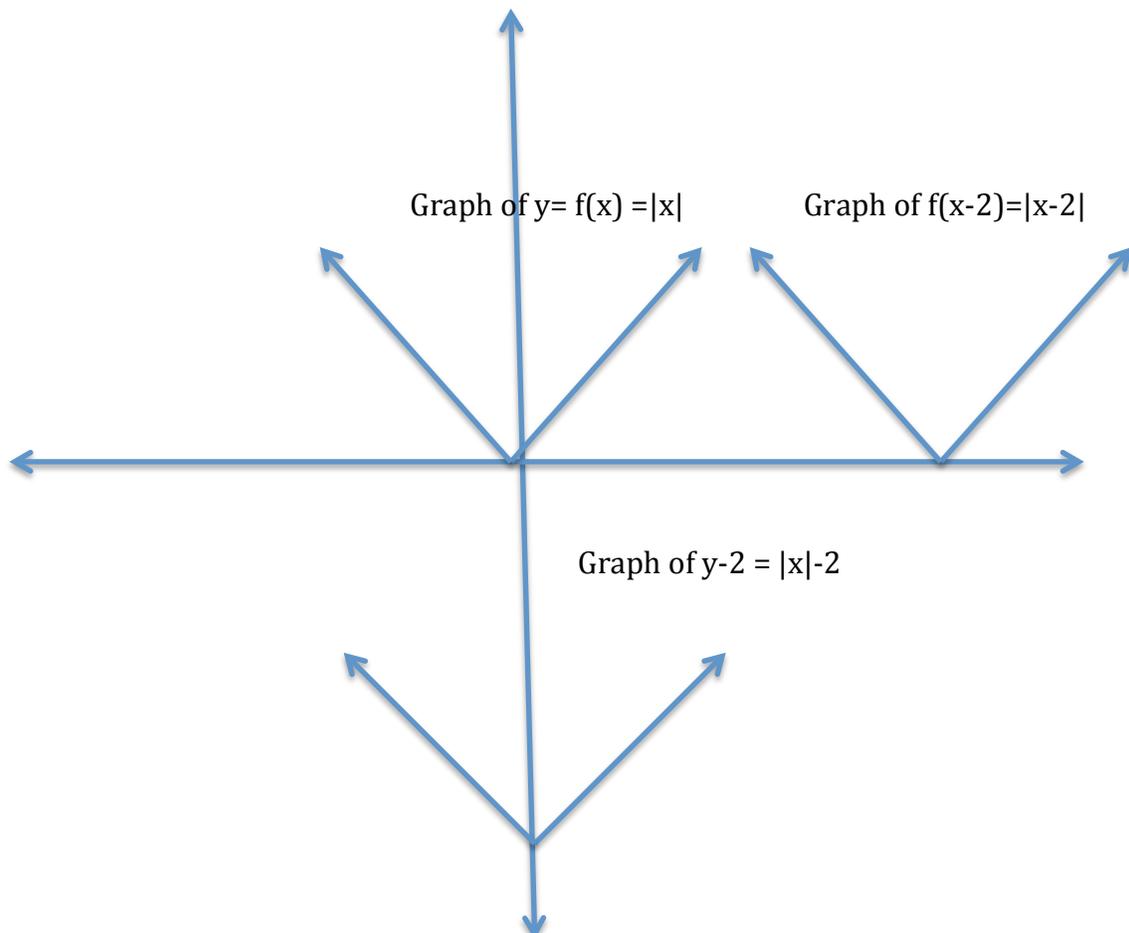
Denoted as $f \circ g(x)$ or $f(g(x))$, in this case you *plug in the value into the first function from the right* and then you *plug in the result or output into the second function from the right*. Basically you go from right to left.

To find the domain of $f \circ g$ find the valid values for the inside function first, say $g(x)$ here, and then find out for what values of x is $g(x)$ in the domain of f .

(Problem 1.3.34) Can you write down $f \circ g(x)$ when $f(x)$ is x^3-2 and $g(x)$ is $1-4x$? What is its domain?

Start by writing $f \circ g(x) = f(g(x))$ to get an idea of what needs to be done.

**Graph of $|x|-2$ and $|x-2|$ starting with graph of $|x|$
Shift right for $|x+2|$ and shift down for $|x|-2$**



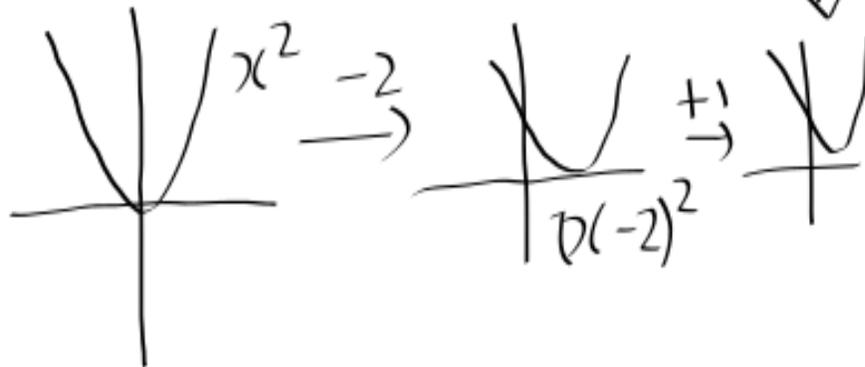
Graph of x^2-4x+4 starting with graph of x^2

Complete the square first to get a square and write it as $(x-2)^2+1$

Now get the desired graph by shifting to right by 2 and moving up by 1.

$$x^2 - 4x + 5 = x^2 - 4x + 4 - 4 + 5$$

$$= (x-2)^2 + 1 \quad \downarrow$$



Problem about composing functions: Given $f(x) = 1/x$, $g(x) = x+1$, find $f \circ g$, $g \circ f$, $f \circ f$, $g \circ g$ and their domains.

$$f \circ g(x) = f(g(x)) = 1/g(x) = 1/(x+1).$$

Its domain will be all real numbers such that $x+1$ is not zero. In other words, all real numbers except -1 .

$$g \circ f(x) = g(f(x)) = f(x)+1 = (1/x)+1.$$

Its domain will be all real numbers other than zero.

$$f \circ f(x) = f(f(x)) = 1/f(x) = 1/(1/x) = x.$$

Its domain will be all real numbers such that other than zero. Even though the final result is x itself, in order to start the process we need to use $1/x$, so we still need to avoid zero.

$$g \circ g(x) = g(g(x)) = g(x)+1 = x+2.$$

Its domain will be all real numbers.