

1. (10 points) If p and q are true, find the truth value of $(p \vee q) \wedge q$.

Solution: $p \vee q$ is true and q is true so $(p \vee q) \wedge q$ is true.

2. (10 points) Write the truth table for $(p \vee q) \wedge q$.

	p	q	$p \vee q$	$(p \vee q) \wedge q$
	T	T	T	T
Solution:	T	F	T	F
	F	T	T	T
	F	F	F	F

3. (20 points) Write the converse and contrapositive of the following statement: "If a natural number N is composite, then some prime number smaller than or equal to \sqrt{N} will divide N ."

Solution: Converse is "If some prime number smaller than or equal to \sqrt{N} will divide a natural number N , then N is composite."

Contrapositive is "If no prime number smaller than or equal to \sqrt{N} will divide a natural number N , then N is prime."

4. (15 points) Write the following statement in logical symbols: "If a person is single and making less than \$20,000 a year or married and making less than \$30,000 a year then they are considered eligible for subsidized housing in the city."

Solution: Let S = A person is single ; L = Income is less than 20000; M = A person is married ; P = Income is less than 30000 ; H = A person is eligible for subsidized housing. Then the given statement can be written as $(S \wedge L) \vee (M \wedge P) \rightarrow H$.

5. (15 points) Prove by contradiction: If there are five people in a party then there are atleast three people of the same gender.

Solution: Suppose not. Then there are at the most two males and two females. Then the total will be less than or equal to 4 people, which is a contradiction.

6. (20 points) Are the following logically equivalent? $(p \vee q) \wedge (q \vee r)$ and $p \vee q \vee r$.

p	q	r	$(p \vee q) \wedge (q \vee r)$	$p \vee q \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Solution: We see from above truth table that in two instances the statements have different truth values, so they are not equivalent.

7. (10 points) Prove the statement p deductively, starting from the statement q , where:

p : The law says you shall not drive more than 55 mph

q : The government gets some of its money from traffic police.

Solution: The law says you shall not drive more than 55 mph. There are many people who drive. Some of them drive over 55 mph sometimes. The police sometimes ticket people when they drive more than 55 mph. So some tickets are written which carry fines. Some of these fines are paid. Thus the government gets some of its money from traffic police.