

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

ANSWERS WITHOUT EXPLANATION WILL ONLY GET 40%

Time Limit 45 minutes; EACH QN 20 POINTS

Please read the questions carefully before answering

It is recommended that you try those problems you are most comfortable with, first.

1. Write the truth value for the following statements, given p : John understood homework problem well, q : John did well in tests , r : John got a good grade, and that p is true, q is true, r is true.

a) $(p \rightarrow q) \rightarrow r$

b) $p \vee q$

c) $p \wedge q$

d) $p \wedge q \wedge \bar{r}$

Soln: (a) p is true and q is true means $p \rightarrow q$ is true. $p \rightarrow q$ is true and r is true means $(p \rightarrow q) \rightarrow r$ is true. So if John understood the hw problems, he would do well in tests and get a good grade.

(b) True (c) True (d) True.

2. Let $P(x)$ be the statement $x - 1 < x^2$. Write the following statements in words and tell whether it is true or false. Justify your answer.

a) For all real numbers, $P(x)$.

b) For no real number, $P(x)$.

$x - 1 < x^2 \Leftrightarrow x^2 - x + 1 > 0 \Leftrightarrow (x - .5)^2 + .75 > 0$. The last expression is a sum of a square and the positive number .75. Hence it is always positive. Since it is equivalent to $P(x)$, $P(x)$ is true for all real numbers.

a) $\forall x \in \mathbf{R}, P(x)$. True

b) $\nexists x \in \mathbf{R}$, such that $P(x)$. False

3. Prove by contradiction and by induction:

For all positive integers n , $2n$ is greater than or equal to $n+1$.

Proof by contradiction. Suppose it is not true. Then there is n such that $2n < n + 1$. But this means $n < 1$ which means if n is not a positive

integer. Thus the contrapositive is true and we have proven the given statement by contradiction.

Proof by induction: For $n = 1$, $2n = 2 \geq 1 + 1 = 2$. Assume $2n \geq n + 1$. Then for $n + 1$, we have $2(n + 1) = 2n + 2 \geq (n + 1) + 2 = n + 3 \geq n + 2 = (n + 1) + 1$.

4. (a) Write down the power set (set of all subsets) $P(A)$ of the set A given by $\{1,2,3\}$. What is the set of all unions and intersections of the elements of $P(A)$?

(b) Convert 3112 to binary.

a) $P(A) = \{ \{ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$. The set of all unions and intersections of the elements of A , i.e, the subsets of A , is also the same as $P(A)$ because union or intersection of any two subsets of A is also a subset of A .

b) $3112 = 110000101000$

5. Given two people a and b , you say aRb (a is related to b) if a is a relative of b . Is this an equivalence relation? (Assume each person is related to himself or herself). Give an argument to prove that if you divide the whole world into equivalence classes based on this relation, there might be only one equivalence class.

It is easy to prove that R is reflexive, symmetric and transitive. If it is true that all human beings came from just one man and a woman, then everyone in the world is related and thus there is only one equivalence class.