9/29/2017 Fall 2017, Calculus I Quiz 4 (version 1) Solutions Sitaraman

Howard University Math Department

1. Find the derivative of $f(x) = \dfrac{x+e^x}{1+x^2}$ using the quotient rule:

   Solution:

   We apply quotient rule first, and then product rule to find $(xe^x)'$.

$$f'(x) = \frac{(x+e^x)'(1+x^2) - (x+e^x)(1+x^2)'}{(1+x^2)^2} = \frac{((x)' + (e^x)')(1+x^2) - (x+e^x)(2x)}{(1+x^2)^2}$$

$$= \frac{(1+e^x)(1+x^2) - (x+e^x)2x}{(1+x)^2} = \frac{1+x^2+x^2e^x+e^x-2x^2-2xe^x}{(1+x^2)^2}$$

$$= \frac{1-x^2+x^2e^x+e^x-2xe^x}{1+2x^2+x^4}.$$

   Note: You CANNOT CANCEL $1+x^2$ here with just one of the terms in numerator because in the numerator we are subtracting. It has to be canceled with EACH term. If it is multiplication in the numerator then you cancel just one term. You can, however, separate this fraction into two fractions and then cancel, as long as you don't have any addition or subtraction in the numerator any more.

2. Find velocity and acceleration of the particle moving at $s(t) = e^2 + \sqrt{t}$ at time t.

   Solution:

   Note that e^2 is a fixed number. So its derivative is zero.

$$v(t) = s'(t) = (e^2)' + (\sqrt{t})' = (e^2)' + (t^{1/2})' = 0 + (1/2)t^{\frac{1}{2}-1} = (1/2)t^{-1/2} = \frac{1}{2\sqrt{t}}.$$

$$a(t) = v'(t) = (1/2)(t^{-1/2})' = (1/2)(-1/2)t^{-\frac{1}{2}-1}$$

$$= (-1/4)t^{-3/2} = \frac{-1}{4t^{\frac{3}{2}}}.$$

3. Find dy/dx if $y = \cot x = \cos x / \sin x$.

   Solution:

   To find derivative we use quotient rule.

   We know $(\sin x)' = \cos x$ and $(\cos x)' = -\sin x$.

We get

$$y' = (\cos x/\sin x)' = [(\cos x)'(\sin x) - \cos x(\sin x)']/(\sin x)^2 = [-\sin x \sin x - \cos x(-\cos x)]/(\sin x)^2$$

$$= -(\cos^2 x + \sin^2 x)/(\sin^2 x) = -1/(\sin^2 x) = -(cosec(x))^2.$$

NOTE: $\sin x^2$ is NOT SAME as $(\sin x)^2 = \sin^2 x$. Here we have the latter.

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$$= \frac{(e^x + xe^x)(1+x^2) - 2x^2e^x}{(1+x)^2} = e^x\left(\frac{1+x+x^2+x^3-2x^2}{(1+x^2)^2}\right)$$

$$= e^x\left(\frac{1+x-x^2+x^3}{1+2x^2+x^4}\right).$$

   Note: You CANNOT CANCEL $1+x^2$ here with just one of the terms in numerator because in the numerator we are subtracting. It has to be canceled with EACH term. If it is multiplication in the numerator then you cancel just one term. You can, however, separate this fraction into two fractions and then cancel, as long as you don't have any addition or subtraction in the numerator any more.

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$$= e^t - (1/4)t^{-3/2} = e^t - \frac{1}{4t^{\frac{3}{2}}}.$$

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