

All problems 20 points. Anything you get over 100 is extra credit. Please give explanations for all your answers. Answers without explanation will only get 40 percent.

1. If  $G$  is abelian then show that  $\phi : G \rightarrow G$  given by  $\phi(g) = g^{-1}$  is an automorphism.
2. Prove that  $\mathbf{Z}_4$  under addition is isomorphic to  $U(5)$  under multiplication via the map  $\phi(x) = 2^x \pmod{5}$ .
3. Show that the map from  $\mathbf{Z}$  under addition to itself given by  $\phi(x) = 2x$  is not an automorphism.
4. If  $\phi$  is an isomorphism from  $G$  to  $\overline{G}$  show that  $\phi(x^{-1}) = (\phi(x))^{-1}$
5. List all the distinct cosets of  $H = \{0, 3, 6, 9\}$  in  $\mathbf{Z}_{12}$  under addition.
6. If  $|G| = 24$  what are all the possible orders of the subgroups of  $G$ ?