

All problems 20 points. Anything you get over 100 is extra credit. Please give explanations for all your answers. Answers without explanation will only get 40 percent.

1. Is the set of rational numbers with odd number denominators a subgroup of the set of all rational numbers under addition? Is it a subgroup under multiplication? (in this case we consider only non-zero rational numbers).
2. Prove if true or give counter-example if false: (a) The union of two subgroups of a group is also a subgroup (b) Every subgroup of an abelian group is abelian.
3. List all the subgroups of $U(20)$. Which of them are cyclic? If they are cyclic, produce a generator.
4. List all the elements of order 10 in \mathbf{Z}_{240} . List the elements of the unique cyclic subgroup of order 10 in the same group.
5. List all the elements (in cycle notation) in the cyclic subgroup generated by (1234) in S_4 . What is the order of this subgroup? What are the orders of the elements of this subgroup?
6. Write down the following permutation as (i) a product of disjoint cycles (ii) a product of 2 cycles. Find its order and its inverse:

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 5 & 4 & 1 & 2 & 3 & 6 \end{array}$$