Patterns in Mathematics

 Final

Dr. McGowan Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

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1. In each of the diagrams below, with the given road system from A to B, how many paths are there from A to B, if the only possible moves are up or to the right?
	1. B

A

* 1. B

A

* 1. Careful! Count! B

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1. The fine that was imposed for the crime of dismantling 90% of the trolley system in 1949 was $5,000. If that money were put in a bank and earned 7% simple interest since then, how much money would there be?
2. Recall that Arrhenius’s law is $∆F= αln⁡\left(\frac{C}{C\_{0}}\right)$, where ∆F is the amount of radiative forcing (extra heat power from the sun), C is the current level of CO2 in the atmosphere (about 390), C0 is the baseline amount of CO2 in the atmosphere (from pre-industrialization – about 275), and α is 6.3. Compute ∆F.
3. Units of energy.
	1. If you burn a 100 Watt light bulb for 250 hours, how many kilowatt-hours (kWh) of energy is this?
	2. Suppose 1 kWh = 3.6(105) J. How many Joules does it take to burn your 100 W light bulb for 250 hours?
4. Suppose City A builds 10 small power generating plants in succession, each producing 20 MW of power, and it takes 10 hours to produce each one. City B builds one large power generating plant, which produces 200 MW of power, and it takes 100 hours to build it. Each city begins building at the same time.
	1. After 20 hours, how much energy have City A’s power plants produced? How much energy has City B’s plant produced?
	2. After 30 hours, how much has each city’s power plants produced?
	3. After 40 hours?
	4. After 100 hours? 110 hours?
	5. 20 0 hours?
5. For each step of the following proof, give the appropriate mathematical law to justify the operation. (Definition of negative exponents, multiplicative inverse, multiplicative identity, distributive law, associative law of multiplication, inverse of product, commutative law of multiplication)

$$\frac{1}{x}+\frac{1}{y}=x^{-1}+y^{-1}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

$$=1x^{-1}+1y^{-1}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

$$=yy^{-1}x^{-1}+xx^{-1}y^{-1}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

$$=(y+x)y^{-1}x^{-1}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

$$=\left(y+x\right)\left(xy\right)^{-1}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

$$=\frac{y+x}{xy}\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

1. Rewrite the sum, using summation notation:

$$\left(1^{2}-1\right)+\left(2^{2}-1\right)+(3^{2}-1)+\cdots +\left(50^{2}-1\right)=\sum\_{\\_\\_\\_\\_\\_}^{\\_\\_\\_\\_\\_}\\_\\_\\_\\_\\_\\_\\_\\_\\_$$

1. Compute the following sums:
	1. $\sum\_{k=1}^{6}3k-1$
	2. $\sum\_{k=1}^{6}3+0∙k$
2. Suppose you have a group of 10 people
	1. How many ways can you choose a committee of 3 people from a group of 10 people?
	2. How many ways can you choose a president, vice president, and secretary from 10 people?
3. How many ways can you rearrange the letters in REITERATE?
4. You borrow $9,000 for a car at 6% interest on a 5-year loan. Recall that the equation for a loan at maturity is

 $P\left(1+\frac{R}{12}\right)^{n}-Pr\frac{\left(1+\frac{R}{12}\right)^{n}-1}{\frac{R}{12}}=0$, where P is principal, R is the annual interest rate, n is the number of months, and r is the proportion of principal paid each month.

* 1. What are your monthly payments?
	2. How much do you owe after 6 months? Use the formula for the amount owed after k months (A(k)):

$$A\left(k\right)=P\left(1+\frac{R}{12}\right)^{k}-Pr\sum\_{j=0}^{k-1}\left(1+\frac{R}{12}\right)^{j}$$

1. Suppose that for each mile of road, the government collects $200,000 in user taxes. Suppose it costs $1,000,000 to build each mile of road. Start with one mile of road in year 0.
	1. After 1 year, this mile of road has generated $200,000. How much more road can be built for this amount? (Hint: the answer is a fraction of a mile.)
	2. After you have added this fraction to the original mile, how much road do you have?
	3. How much user taxes will 1.2 miles of road generate in year 2?
	4. How much additional road can be built for $240,000? (This will again be a fraction of a mile.)
	5. How many total miles of road are there, after this addition is built? (Add the fraction obtained in part c to the 1.2 miles of road from part b. )
	6. Continue in the same fashion. After 10 years, how many miles of road will there be, and how much gas tax will it generate?
2. Consider the following arguments.  If necessary, rephrase the first premise so that is has the form all S are P.  Draw a Venn diagram to determine whether the argument is valid.  Discuss the truth of the premises and state whether the argument is sound.
	1. First Example:
		1. Premise:  All European countries use the euro as currency.
		2. Premise:  Great Britain is a European country
		3. Conclusion:  Great Britain uses the euro as currency.
	2. Second Example:
		1. Premise:  All dairy products contain protein.
		2. Premise:  Soybeans contain protein.
		3. Conclusion:  Soybeans are dairy products.
3. Suppose a 1-square-meter panel of solar cells with 100% efficiency produces 1000 Watts of power. If a solar panel has an efficiency of 20% and receives the equivalent of 6 hours of direct sunlight per day.
	1. How much energy, in joules, can it produce each day? (1 Watt = 1 Joule/sec; 60 Watts \* 10 sec = 600 Joules)
	2. What average power, in watts, does the panel produce? (Consider what fraction of the day 6 hrs is.)
	3. Suppose you want to supply 1 kilowatt of power to a house ( the average household power requirement by putting solar panels on its roof. For the solar cells described in exercise 1, how many square meters of solar panels would you need? Assume that you can make use of the average power from the solar cells (by, for example, storing the energy in batteries until it is needed).
4. If it takes 10 minutes to shower, and 7000 people share 20 showers in a gym, to which they have access for 15 hours a day, compute the shower-demand: number of people times amount of time per shower; and the shower-availability: the number of shower-hours available per day. How often may each person shower?