Solutions to Selected Assignment Problems, 11/27/2015

1. Say which of the following is a geometric sequence and find the sum of the first 10 terms of the ones which are geometric:

 a) 3, 9, 15, 21,26,…

 b) 1, 4,16,64,…

a) is neither arithmetic nor geometric. Although it looks like it is arithmetic at first glance, with each term going up by 6 from the previous term, you see that the difference between 21 and 26 is 5 and not 6.

b) This is a geometric sequence that is actually powers of 4 : 1, 41,42,43,…

The sum is given by the formula a(1- rn)/(1-r) where a is first term and r is common ratio and n is number of terms. Here a = 1, r = 4, n = 10. So the sum of first 10 terms is 1(1-410)/(1-4) = 349525.

2. Suppose everyone in a community spends 80% of what they have, locally. Starting with an amount of P, say the first person spends 0.8P, then 80% of the 0.8P is spent and so on. How much is spent after 10 cycles of spending? What is the multiplier effect (after an “infinite” number of cycles of spending)?

Look into your notes (or Walter book section 20.2) to see how this works.

After 10 cycles the total will be 0.8P + 0.82P + 0.83P +…..+0.810P = 0.8P x (1 – 0.810) / (1- 0.8)

 (geometric sequence with a = 0.8P and r = 0.8)

As the number cycles get bigger, the term 1- 0.8n in the numerator gets close to 1 because the powers of 0.8 get smaller and smaller. So as the sum “goes to infinity” (which really in this case means as we add more and more terms and the number of terms gets very big) it gets closer to

 0.8P / (1- 0.8) = (0.8) x P / (0.2) = 4 x P (because 0.8 / 0.2 = 4)

4 times the original amount P is spent after a large number of cycles.

Now the first person spends 0.8P. So the multiplier effect is that 5 times the amount spent by the first person, namely 0.8P, is spent in the community because 4P = 5 x 0.8P

3. Repeat problem 3 for savings, assuming the first person saves 20% of P and everyone else saves 20% of what they get.

Look into your notes or Walter section 20.2 to see how this works.

After 10 cycles the total will be 0.2P + 0.2 x (0.8P) + 0.2 x (0.82P) + 0.2 x (0.83P) +…..+ 0.2 x (0.89P) = 0.2P x (1 – 0.810) / (1- 0.8)

(geometric sequence with a = 0.2P and r = 0.8 ; last term has 0.89 because the first term starts without 0.8 as opposed to sum in previous problem)

The “infinite sum” in this case is : a/(1-r) = 0.2P / (1-0.8) = 0.2P / (0.2) = P

So starting with 0.2P being saved by first person we end up with P, for a multiplier of 5, because

 P = 5 x 0.2P

4. Starting with X tonnes of virgin pulp per day, what should X be in the above system to maintain a daily production of 1500 tonnes of paper products if the recycling rate of easy to recycle paper is $3/4 ?$ Assume everything else as in Chapter 20 section 3 (or what we did in class) : 3/4 of pulp is made into paper and so on. Go through the process as in section 3 and you will get X times a factor. Set this equal to 1500 and solve for X.

 

Look into your notes or Walter section 20.3 to see how this works.

Basically you replace 2/3 with 3/4 in the discussion on recycling in 20.3.

After the first day 3/4 of the paper gets recycled, and since the paper used is 3/4 of the X amount of pulp, the amount of pulp (coming from the recycled paper) is 3/4 x 3/4 x X = 9/16 X.

The total amount of pulp for the second day will be X amount of new pulp plus 9/16 times X amount of pulp from recycled paper for a total of X + (9/16)X.

For the third day it will be X + (9/16)(pulp from second day) = X + (9/16) ( X + (9/16) X )

 = X + (9/16)X + (9/16)2X

and so on. The “infinite” sum will look like X + (9/16)X + (9/16)2X + (9/16)3X +….

 = X / (1 – (9/16) ) = X x (16/7)

Here we used the formula a/(1-r) for the infinite sum with a = X and r = 9/16.

 So if the daily supply of new pulp is X then after many days the total supply every day

Will be approximately 16/7 times X. We want this to equal 1500 because that is how much pulp is needed per day. So set the m equal and solve for X:

 1500 = (16/7)X gives X = (7/16)(1500) = 656.25 tonnes.

5. Look at the table for the July 2015 temperature readings for DC in the Environmental Statistics handout on update page. What percentile value do you assign to the 85 degree temperature reading ?

These readings are in a table on page 16 of the handout. 85 appears as the 24th, 25th, 26th and 27th values. So we take it to be the 23+ (4/2) = 25th value. So its percentile rank is 25/31 = 0.8064 = 81% approximately which gives 81st percentile.

6. One incoming Howard University freshman was ranked 10th in a class of 125; another ranked 75th in a class of 620. Which has the higher percentile rank?

10th rank of 125 means 115 students are below this score. So percentile rank is

115/125 = 92nd percentile

75th of 620 means rank is 545/620 = 88th percentile.

So 10th of 125 is the higher percentile rank. You see how percentile ranking helps to compare ranks from different size populations.

7. A box has 10 white socks and 10 black socks. If a pair is picked up at random, what is the probability that they are both white? What is the probability that they match (i.e, are both the same color) ?

For all probability problems start with the basic formula

Probability = Number of favorable outcomes / Total number outcomes.

Here the numerator is the nuber of ways to pick 2 white socks from 10. Order does not matter, so we use the formula 10C2 = (10 x 9) / 2!

The denominator is, similarly, 20C2 = (20 x 19) / 2!

When you divide the 2! cancels out and we get (10 x 9) / (20 x 19) = 9 / 38.

Probability that they are of same color is probability of either them being both white or both black. In this case in the numerator we get 10C2 + 10C2 = 2 x 10C2.

The denominator will be same. So answer will be 2 x (9/38) = 9/19.

You can also get this by adding probability of getting both white and probability of getting both black (both will equal 9/38). When finding probability of EITHER of two events happening, you can add their probabilities if they are mutually exclusive events.

8. What is the probability that out of five people, at least 2 have the same birthday?

 This is 1 – P(A’) where A’ is the opposite of A, in this case at least 2 having the same birthday. So A’ is the event that no two have same birthday, or that all five have different birthdays.

 P(A’) = 365P5 / 3655. The numerator is 365P5 because it is the number of ways 5 people can have different birthdays out of 365 days. It is like making 5 people sit in 5 of 365 places. The first person can choose any one of 365, the second has to choose from remaining 364 (because they all have to be different) and so on.

 So it is 365 x 364 x 363 x 362 x 361 = 365P5.

 The denominator is 3655 because we are just looking at all the ways 5 people can have birthdays, so repetition is allowed. Each person could be born in any one of 365 days and so you just multiply 365 by itself five times.

 Check that 1 – (365P5 / 3655) = 1 – 0.9729 = 0.0271 or 2.71%

 9. (units of energy pdf on Update page) From the page 15 table titled “Units of Energy and Power) you get that one barrel of oil produces 6118 GJ (gigajoules) of energy. If a solar panel is capable of producing 200 Watts of power, how many such panels sitting in the sun for 100 hours each are needed to produce the same amount of energy as one barrel of oil? All the information you need is in the same table.

 One GigaJoule is 109 Joules.

 So 6118 GJ is 6118 x 109 Joules.

 200 Watts for 100 hours each is 20,000 Watt-hours total.

 One Watt is one Joule / sec, in one hour we get 60 x 60 = 3600 Joules, so one Watt for 100 hours gives 3600 x 100 Joules or 36 x 104 Joules.

 So 200 Watts for 100 hours is 200 x (36 x 104) = 2 x 100 x 36 x 104 = 72 x 106 Joules.

 Now we are ready to get final answer. We just need to see how many of 72 x 106 is in

 6118 x 109.

 This is 6118 x 109 / (72 x 106) = (6118 / 72) x 103 = 84.972 x 103 = 84972 panels.

 FYI : By the way an array of about 85000 panels each of 200 Watts produces about 19000 Kilowatts or 19 megawatts. We have now solar farms that produce 100 or even 1000 Megawatts.