

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 100 minutes

Please read the questions carefully before answering

Challenge problem is extra credit 20 points.

Total 100 points.

1. (40 points) Find the derivatives of the following functions:

(a) $x^3 e^x$ (b) $\sin(3x^2 + 1)$ (c) $\frac{x + e^x}{x}$ (d) $\cos^{-1}x + \ln(x^2 + 1)$

2. (20 points) Use implicit differentiation to find dy/dx given that $xy^2 + y^2 - x = 0$.

3. (20 points) The population of a town after t years is given by the formula $P(t) = 3t + 5t^{3/2} + 6000$. Using differentials, estimate the change in the population during a period of 3 months (0.25 years) at the end of 5 years. (i.e from $t = 5$ to $t = 5.25$.)

4. (20 points) Find the equation of the tangent line to the graph of $f(x)$ where $f(x) = y = \frac{x-1}{x+1}$ at a general point $x = a$. Find the points where it has a horizontal tangent line or the slope of the tangent is undefined.

5. Find the derivative of $f(x) = (x^2 + x + 1)(x^3 - 2x)$ using product rule.

6. Given that $f(1) = 1$, $f'(1) = -1$, $g(1) = 2$, and $g'(1) = 1$, find the derivative of $f(x)g(x)$ at $x = 1$.

7. Find the derivative of $\sin(x^2)$ and $(x^2 + 1)^{10}$ using chain rule.

8. Using implicit differentiation [WITH RESPECT TO y] find $\frac{dx}{dy}$ given that $\cos x = y$. Write your answer so that $\frac{dx}{dy}$ is in terms of y . Using this find the derivative of $\cos^{-1}y$.

9. Differentiate using logarithms: $\frac{x^{\sin x}}{(x^2+1)^{1/3}}$.

10. A rocket is at a height of $h(t)$ at time t seconds. An observer from a distance of 1000 meters measures the angle of elevation as θ at time t . Find the velocity of the rocket after 10 seconds if the angle of elevation is $\pi/4$ radians and changing at 0.1 radian per second.

11. Find the critical points of $x^{1/3}(x+1)$. Explain whether they are relative maxima or minima using first or second derivative tests.

12. An object is moving such that its position at time t is given by $s(t)$. State whether the following are true or false. If true explain why and if false disprove it or provide counterexample: (a) The speed is given by $s'(t)$. (b) The acceleration is given by $s''(t)$. (c) The velocity is always positive. (d) The distance traveled from time t_1 to t_2 is given by $s(t_2) - s(t_1)$.

13. The population of a town after t years is given by $P(t) = \frac{1200}{1 + e^{-0.1t}}$. Find the rate of change of this population after 5 years. Is it always increasing or decreasing? What happens to the population as t approaches ∞ ?

(CONTINUED ON NEXT PAGE)

14. Find the differential of $f(x) = x^{\frac{1}{3}}$. Use your result to estimate $126^{\frac{1}{3}}$.
15. Two cars leave from the same intersection. A goes north and B goes east. A is moving at 10 mph and car B is moving at 20 mph after 3 hours.. Find the rate of change of the distance between them, at that time, given that A and B are both 100 miles away from the intersection, *at that time*. (This is the same as the speed of one car relative to the other at that instant).
16. Find the critical points of the function $y = xe^{-x^2}$.
17. (20 points) Show that the absolute maximum is 123 and absolute minimum is -37 for $f(x) = -2x^3 + 3x^2 + 12x - 5$ in the interval $-4 \leq x \leq 4$. You must find all critical points.