11/11/2015 Fall 2015, Calculus I Quiz 7 Sitaraman

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Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.

1. (20 points) Graph the function $y = x^3 - x^2$.

   You must find the following:

   a) All the local maxima and minima and inflexion points.

   b) Where it is increasing, decreasing, concave up and concave down.

   Solution:

   We have $y' = 3x^2 - 2x$. This is defined everywhere but it is zero when $3x^2 - 2x = 0 \implies x(3x-2) = 0 \implies x = 0$ or $x = 2/3$. So 0 and 2/3 are the only critical points.

   $y'' = 6x - 2$ This is zero at $x = 1/3$. It is negative (= -2) when $x = 0$ and positive (= 2) when $x = 2/3$.

   So it has an infexion point at 1/3 and a local maximum at 0 and a local minimum at 2.

   Also $x^3 - x^2 = x^2(x-1) = 0$ when $x = 0$ or $x = 1$.

   Looking at the highest degree term we see that x^3 goes to negative infinity as x goes to $-\infty$ and goes to ∞ as $x \to \infty$.

   $y'' = 6x - 2$ is negative to left of 1/3 and positive to the right of 1/3 – graph of $6x - 2$ is a straight line that crosses x axis at 1/3. In between $-\infty$ and 1/3 it is concave down because y'' is negative and from 1/3 onwards it is concave up. The function has a point of inflexion at 1/3 where it changes concavity.

   So the graph starts in the bottom left corner ("negative infinity"), goes up (increases) to 0, turns around, it is decreasing between 0 and 2/3, starts bending up at 1/3, turns around at 2/3, increasing from 2/3 onwards, crosses x axis at 1 and then keeps going up (increasing).

2. (extra credit 10 points) Using the mean value theorem show that the function $f(x) = 2x + \sin x$ has at the most one zero, i.e, not more than one value of x for which $2x + \sin x = 0$.

   Solution: The derivative $f'(x) = 2 + \cos x$ is never zero, because $\cos x = -2$ is impossible. Thus by mean value theorem, there can only be one zero. Otherwise, function would have a turning point and the derivative would be zero at that point.

   Actually for this problem you could use Rolle's theorem as well.

3. (10 points) Show that the mean value theorem works for e^x in [0,1]. First verify it satisfies MVT conditions there.

   Solution: e^x is continuous and differentiable everwhere, so it satisfies MVT conditions in [0,1]

   We want $f'(c) = (e^1 - e^0)/(1 - 0) = e - 1$ for some $c \in (0, 1)$.

   In other words, $e^c = e - 1$.

   Taking ln of both sides, we get $ln(e^c) = ln(e - 1) \implies c = ln(e - 1) = 0.54$ which is in (0,1)