10/28/2015 Fall 2015, Calculus I Quiz 6 Sitaraman

Howard University Math Department

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points.

1. Given that a radioactive material decays according to the function $N(t) = N_0 e^{-kt}$ find the formula for Half-life, namely time for the original amount N_0 to be reduced in half. You can leave your answer in terms of k.

$$N(t) = N_0 e^{-kt} = \frac{1}{2} N_0 \implies e^{-kt} = \frac{1}{2}$$

$$\text{(Take ln of both sides)} \implies -kt = ln(1/2) = ln1 - ln2 = -ln2 \implies t = ln2/k.$$

2. The marginal cost of producing an item when a items are produced is the cost of producing one more item. In the language of differentials, the marginal cost is $dC = \frac{dC}{dx} dx$ when $dx = 1, x = a$. In other words, marginal cost when a items are produced equals $C'(a)$. Here $C(x)$ is the cost of producing x items. Find the marginal cost of an item whose cost function is given by $C(x) = x^3 + 100x^2 + 245$ when 100 items are being produced.

   Solution:

   $dC = C'(100) = 3x^2 + 100(2x)$ with $x = 100$. So $C'(100) = 3(100^2) + 200(100) = 50,000$.

3. Volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$. Find the rate at which the volume of a ball is increasing when the radius is increasing at 1cm/sec and the radius is 10 cm.

   Solution:

   $V(r) = \frac{4}{3}\pi r^3$.

   Differentiating both sides with respect to r, we get $V'(t) = V'(r)(r'(t)) = \frac{4}{3}\pi(3r^2)r'(t) = 4\pi r^2 r'(t)$.

   Here we used chain rule.

   Putting $r = 10, r'(10) = 1$, we get $V'(10) = 4\pi(10)^2(1) = 400\pi$ cc (cubic cm) per sec.

4. This problem is extra credit but it is easy.

   (6 points) Find the linearization (local linear approximation) of $y = (1 + x)^4$ at $x = 0$.

   (4 points) Use it to approximate 1.01^4 and compare with actual value.

Solution:

The derivative of $(1+x)^4$ can be found using chain rule as $f'(x) = 4(1+x)^3(1) = 4(1+x)^3$.

We have $f(x) = f(0) + f'(0)(x-0) = (1+0)^4 + 4(1+0)^3(x) = 1+4x$.

Using this formula, $(1.01)^4 = 1 + 4(.01) = 1.04$ approximately. Here $x = 0.01$ because we want $1 + x = 1.01$.

Comparing with actual value, $(1.01)^4 = 1.0406$.

You can see that it is a good approximation.