

**Howard University Math Department**

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

Each problem 10 points.

1. Given  $f(x) = \sin x$ ,  $g(x) = \sqrt{x}$ , find the following:

(a)  $f \circ g(x)$

(b)  $f \circ g(0)$

(c) The domain of  $f \circ g$

Solution:

1a.  $f \circ g(x) = f(g(x)) = \sin(\sqrt{x})$ .

1b. Plug in 0 in 1a to get  $\sin(\sqrt{0}) = \sin(0) = 0$ .

The domain of  $f \circ g$  will contain those values of  $x$  in the domain of  $g$  for which  $g(x)$  is in the domain of  $f$ .

The domain of  $g$  is all non-negative real numbers.

So we need to check for a given non-negative real number  $x$  whether  $g(x) = \sqrt{x}$  is in the domain of  $f$ .

The domain of  $f$  is all real  $x$ .

So no matter what the value of  $\sqrt{x}$  we can plug it into  $f(x)$ . So the answer is  $\{x \mid x \geq 0\}$  also written as  $[0, \infty)$ .

2. Let

$$f(x) = \begin{cases} 1 + x, & x < 1, \\ x^2 + 1, & x \geq 1. \end{cases}$$

(a) Find  $f(-1)$ ,  $f(1)$ ,  $f(2)$ .

(b) Draw the graph of  $f(x)$ .

Solution:

Use the equation  $y = 1 + x$  upto but not including  $x = 1$  and then  $y = x^2 + 1$  from  $x = 1$ .

$$f(-1) = 1 - 1 = 0; f(1) = 1^2 + 1 = 2; f(2) = 2^2 + 1 = 5.$$

The graph will be a line passing through (0,1) and stopping at (1,2). From (1,2) onwards it will be a part of a parabola facing up.

3. An amount of  $A(0) = 1000$  dollars is invested at 10 percent and interest is compounded annually.

(a) Find  $A(10)$  the amount after 10 years.

(b) Find the doubling time by plugging in the expression for  $A(t)$  in the equation  $A(t) = 2A(0)$  and taking logarithms of both sides.

(c) How long would it take for an amount of 1000 dollars to grow to 4000 dollars? Use only doubling time from (b).

Solution:

(a) 10 percent is 0.1. So  $r = 0.1$ . So  $A(t) = A(0)(1 + r)^t = A(0)(1 + 0.1)^t = A(0)1.1^t$ .  
 $A(10) = 1000(1.1)^{10} = 1000(2.59374) = 2593.74$  dollars.

(b) Doubling time: To get to 2000, we have  $1000(1.1)^t = 2000$  which gives  $1.1^t = 2$ .

Taking logarithms of both sides,  $\log 2 = t \log(1.1)$  which means  $t = \log 2 / \log 1.1 = 7.27$  years or about 7 years, 3 months and 7 days approximately.

(c) 2000 will grow to 4000 in another 7.27 years, so totally 14.54 years or 14 years, 6 months and 15 days approximately.