

Instructions:

NO CALCULATORS

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 45 minutes

Please read the questions carefully before answering

Each problem 20 points unless otherwise stated.

Any points you get in excess of 100 is extra credit.

1. Find the standard form equation of the circle with a diameter having endpoints $(0,0)$ and $(6,8)$. What is the radius of this circle? [Hint: You can find the center using midpoint formula]

Soln: The center is given by the midpoint formula with $(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (6, 8)$:

$$(h, k) = \left(\frac{0+6}{2}, \frac{0+8}{2} \right) = (3, 4).$$

So the equation is of the form $(x - 3)^2 + (y - 4)^2 = r^2$ where r is the radius.

Since it passes through $(0,0)$ we can plug in $x = 0$ and $y = 0$ into this equation to get $(-3)^2 + (-4)^2 = r^2$. Thus $r^2 = 9 + 16 = 25$ and $r = \sqrt{25} = 5$.

Its equation in standard form is $(x - 3)^2 + (y - 4)^2 = 25$.

Just FYI, its general form equation is $x^2 - 6x + y^2 - 8y = 0$.

You can also find the radius by using distance formula to find the distance between $(3,4)$ and $(0,0)$: $r = \sqrt{(3-0)^2 + (4-0)^2} = \sqrt{25} = 5$.

[Or use distance formula between $(0,0)$ and $(6,8)$ to find diameter, then divide by 2 to get radius].

2. Find the equation of the line parallel to $y - 2x + 1 = 0$ and passing through $(1,1)$.

Soln: First find the slope of the parallel line: $y - 2x + 1 = 0 \implies y = 2x - 1$.

So the slope is 2 for both lines because they are parallel.

So the equation of parallel line is $y = 2x + b$.

We need to find the y -intercept b .

It passes through $(1,1)$ means we can plug in $(1,1)$ into the equation and it will be satisfied.

So we get $1 = 2(1) + b \implies b = 1 - 2 = -1$.

So the desired equation is $y = 2x - 1$.

Note: the two lines are the same!

3. Police use skid marks of tires to estimate the speed of a car at the time of braking. The stopping distance D of a car after brakes are applied varies directly as the square of the speed s .

(a) (8 points) Write an equation that expresses this variation.

(b) (6 points) Find the constant of proportionality k if a car traveling at 50 mph can stop in 240 feet.

(c) (6 points) Find the speed if it stops in 160 feet. You can leave your answer as a square root.

Soln:

(a) The equation is $D = ks^2$.

(b) Putting $D = 240, s = 50$, we get $240 = k(50)^2 \implies k = 240/2500 = 12/125$.

So the equation is $D = \frac{12}{125}s^2$.

(c) Putting $D = 160$ we get $160 = \frac{12}{125}s^2 \implies s^2 = 160 \times 125/12 = 40 \times 125/3 \implies s = \sqrt{\frac{5000}{3}}$.

We only take the positive square root because speed is positive.

4. For the function $y = f(x) = (x - 1)(x + 2)(x - 3)$ find the following:

(a) (6 points) $f(1), f(2), f(-1)$

(b) ((6 points) Find the x intercepts.

(c) (4 points) Find the y intercepts.

(d) (4 points) Check for symmetry about y -axis.

Soln:

a) $f(1) = (1 - 1)(1 + 2)(1 - 3) = (0)(3)(-2) = 0$ because 0 times anything is 0.

$f(2) = (2 - 1)(2 + 2)(2 - 3) = -4$. $f(-1) = (-1 - 1)(-1 + 2)(-1 - 3) = (-2)(1)(-4) = 8$.

b) Put $y = 0$ to get $(x - 1)(x + 2)(x - 3) = 0 \implies x - 1 = 0$ or $x + 2 = 0$ or $x - 3 = 0$ which gives $x = 1, -2, 3$ as the x -intercepts.

c) Put $x = 0$ to get $y = (0 - 1)(0 + 2)(0 - 3) = (-1)(2)(-3) = 6$.

d) We have $y = f(x) = (x - 1)(x + 2)(x - 3) \implies y = x^3 - 2x^2 - 5x + 6$. Put $-x$ in place of x . You get $y = (-x - 1)(-x + 2)(-x - 3) = -x^3 - 2x^2 + 5x + 6$ which is different from the original equation. So it is not symmetric about the y -axis.

5. For the functions $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x - 1}$ find the following:

(a) (6 points) The domains of f and g .

(b) ((8 points) Find $(g \circ f)(1/5)$ and $(f \circ g)(x)$.

(c) (6 points) Find $(f + g)(1)$ and $(fg)(10)$.

Soln:

a) The domain of f is all real numbers other than 0 because we only need to make sure denominator is not zero. The domain of g is all real numbers x such that $x - 1$ is non-negative because all we need to worry about is that we don't need to take the square root of a negative number. We get $x - 1 \geq 0 \implies x \geq 1$. So the domain of g is all real numbers bigger than or equal to 1.

b) $g \circ f(1/5) = g(f(1/5)) = g(1/(1/5)) = g(5) = \sqrt{5 - 1} = 2$.

$f \circ g(x) = f(g(x)) = f(\sqrt{x - 1}) = 1/\sqrt{x - 1}$.

c) $f + g(1) = f(1) + g(1) = (1/1) + (\sqrt{1 - 1}) = 1 + 0 = 1$.

$fg(10) = f(10)g(10) = (1/10)(\sqrt{10 - 1}) = 3/10$.

6. (extra credit) A farmer has 2400 feet of fencing to fence a rectangular horse corral.

(a) (10 points) Find a function that models the area in terms of the width.

(b) (10 points) Find the length and width that maximize the area. What is the maximum area?

Soln:

6a. let w denote the width. if the length is l then $2400 = 2(l + w) \implies 1200 = l + w$. So $l = 1200 - w$ and the area function is $A(w) = w \times l = w(1200 - w)$.

6b. The graph of $A(w) = w(1200 - w) = 1200w - w^2$ is a parabola facing down. So it will have a maximum at its vertex.

The vertex is given by $h = -b/2a = -1200/(2(-1)) = 600$.

So when the width is 600 the area is maximum, and it equals $A(600) = 600(1200 - 600) = 600^2 = 360000$ square feet. (About 9 acres).