Instructions:
NO CALCULATORS
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 45 minutes
Please read the questions carefully before answering
Each problem 20 points unless otherwise stated.
Any points you get in excess of 100 is extra credit.

1. Solve the following equations for x :

   (a) $6x - 5 = 4 - x$

   (b) $\frac{2}{x+3} - \frac{5}{x+1} + 2 = 0$

   Soln:

   1a.

$$6x - 5 = 4 - x \implies 6x + x = 4 + 5 \implies 7x = 9 \implies x = 9/7.$$

   1b. First multiply all by LCM:

$$\frac{2}{x+3} - \frac{5}{x+1} + 2 = 0 \implies (x+1)(x+3)\left(\frac{2}{x+3} - \frac{5}{x+1} + 2\right) = 0$$

   The right hand side stays zero because anything times zero is zero. We get:

$$2(x+1) - 5(x+3) + 2(x+1)(x+3) = 0 \implies 2x + 2 - 5x - 15 + 2(x^2 + 4x + 3) = 0$$
$$\implies 2x^2 + 5x - 7 = 0 \implies (2x+7)(x-1) = 0 \implies x = -7/2 \text{ or } 1.$$

   MUST CHECK TO SEE IF THESE VALUES MAKE DENOMINATOR ZERO

   Neither of these values makes the denominator of original equation zero. So both are solutions. Check your answer in the original equation.

2. John takes twice as long as Jim to paint a room.They take $\frac{10}{3}$ hours to finish it if they worked together. How long does Jim take to paint it by himself?

   Soln: Let x be the time taken by Jim working by himself.

   Then $2x$ is the time taken by John.

   10/3 is the time taken by them to finish it together.

   Start by looking at how much work they would do in 1 hr by taking reciprocals.

   Then in hour they would together do $\frac{3}{10}$ of the work.

   John would paint $\dfrac{1}{2x}$ of the room in 1 hour.

   Jim would paint $\dfrac{1}{x}$ of the room in 1 hour.

   Together they would paint $\dfrac{1}{2x} + \dfrac{1}{x} = \dfrac{3}{2x} = \dfrac{3}{10}$ in 1 hour.

   Taking reciprocals again we find that together they would take $\dfrac{2x}{3} = \dfrac{10}{3} \implies x = 5$ hours

3. Solve by completing the square: $x^2 + 2x - 5 = 0$.

   Soln:First we keep everything involving x on the LHS.

   We get $x^2 + 2x = 5$.

   Next we take half of the $x-$coefficient. $\dfrac{B}{2} = \dfrac{2}{2} = 1$.

   We add the square of $\dfrac{B}{2}$ to both sides.

   We get $x^2 + 2x + 1^2 = 5 + 1^2 \implies (x+1)^2 = 6 \implies x + 1 = \pm\sqrt{6} \implies x = -1 + \pm\sqrt{6}$.

   If you leave the answer like that it is okay.

   So the solution is $-1 + \sqrt{6} = 1.45$ and $-1 - \sqrt{6} = -3.45$.

4. The length of a rectangular garden is 4 feet more than the width. The area is 10 square feet. Find the width.

   Soln:

   Let w be the width. Then $w + 4$ is the length. The area is $(w+4)w = 10$.

   We get

$$w^2 + 4w - 10 = 0 \implies w = \frac{-4 \pm \sqrt{4^2 - 4(1)(-10)}}{2} = \frac{-4 \pm \sqrt{56}}{2}$$

$$= \frac{-4 \pm \sqrt{4(14)}}{2} = \frac{-4 \pm 2\sqrt{14}}{2} = -2 \pm \sqrt{14}.$$

   Since width is positive only $\sqrt{14} - 2$ will work and that is the solution. If you leave it like that that is enough but just FYI width equals $\sqrt{14} - 2 = 1.74$ feet and the length will be 5.74 feet.

5. Solve for x from $4x - 4\sqrt{x-1} = 3$ by first converting it to a quadratic equation. [Note: resulting quadratic equation will be a perfect square, i.e, of the form $(mx+n)^2$].

   Soln: First isolate the square root to one side:

   We get $4\sqrt{x-1} = 4x - 3$.

   Squaring both sides we get $16(x-1) = 9 - 2(3)(4x) + (-4x)^2$.

   YOU CANNOT DO THIS BY SQUARING EACH TERM!!

   For example, square of LHS of original equation is NOT equal to $16x^2 - 16(x - 1.)$

   In general, when addition is involved you cannot square $A + B$ and say it equals $A^2 + B^2$. Only when you multiply or divide it works: $(AB)^2 = A^2B^2$ etc.,

   Simplifying we get the quadratic equation $16x^2 - 40x + 25 = 0$.

   This is a perfect square. In fact, $16x^2 = (4x)^2, 25 = 5^2, 40x = 2(4x)(5)$, so this is simply $(4x-5)^2$.

   So we have $(4x-5)^2 = 0 \implies 4x - 5 = 0 \implies x = 5/4 = 1.25$.

   Since there is only one solution, it should work. You can check by plugging into original equation if you wish:

   $4(5/4) - 4\sqrt{(5/4) - 1} = 5 - 4\sqrt{1/4} = 5 - 2 = 3$.

6. (Optional, 20 points extra credit) Say true or false. Explain why.

   (a) (10 points) If the discriminant $b^2 - 4ac$ equals zero, then the quadratic equation $ax^2 + bx + c$ has exactly one real solution.

(b) (10 points) The only value of x that satisfies $|x+1| \leq 3$ is 2.

Soln:

6a. True. You can see this from the quadratic formula. If discriminant is zero, then the solution is $(-b \pm \sqrt{0})/(2a) = -b/2a$ which is just one number. Note that $\pm\sqrt{0} = 0$.

6b. False. The solution is given by all numbers in the range $-3 \leq x+1 \leq 3 \implies -4 \leq x \leq 2$.