9/14/2012 Fall 2012, Calculus I Quiz 3 Sitaraman

Howard University Math Department

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 10 points. Challenge problem is extra credit.
Any points you get in excess of 40 is extra credit.

1. Find the following limits.

   (a) (8 points) $\lim_{x\to 0} \frac{sinx}{xcosx}$ (Do not use graphs)

   (b) (6 points) $\lim_{x\to -2^+} \frac{x}{x+2}$ (Use any method)

   Solution:

   1a. As shown in class, $\frac{sinx}{x} \to 1$ as $x \to 0$. So we can split the limit as $\lim_{x\to 0} \frac{sinx}{xcosx} = \left(\lim_{x\to 0} \frac{sinx}{x}\right)\left(\lim_{x\to 0} \frac{1}{cosx}\right) = 1(1/cos0) = 1$

   1b. As $x \to -2^+$, say $x = -2+\delta$, where δ is a small positive quantity that goes to zero. Then $x+2 = \delta$ and $\frac{x}{x+2} \to \frac{-2+\delta}{\delta}$, Here the numerator goes to -2 while the denominator goes to 0, so the result is that the fraction goes to negative infinity. (So really there is no limit).

   If you graph this you can see that the function goes to negative infinity as x approaches -2 from the right.

2. Let

$$f(x) = \begin{cases} 3-x, & x < 2, \\ x+2, & x \geq 2. \end{cases}$$

   (8 points) Find $\lim_{x\to 2^+} f(x)$ and $\lim_{x\to 2^-} f(x)$. Does $\lim_{x\to 2} f(x)$ exist? If so, what is it? If not, why not?

   (6 points) Where is the function $f(x)$ continuous and where is it discontinuous?

   Solution:

   To find $\lim_{x\to 2^+} f(x)$ we need to look at what happens to the right of 2.

   Here the function is defined by $x+2$ which is a continuous function and approaches 4 as x approaches 2. [Since it is continous the value is obtaind by just plugging in 2]. So the limit is 4.

To find $\lim\limits_{x \to 2^-} f(x)$ we need to look at what happens to the left of 2.

Here the function is defined by $3-x$ which is also a continuous function and approaches 1 as x approaches 2. As before, since it is continous the value is obtaind by just plugging in 2. So the limit is 1.

Since the two limits are different, $\lim\limits_{x \to 2} f(x)$ does not exist.

From above, we see that it is not continuous at 2 because the limit does not even exist there. But since it is defined by continuous functions $3 - x$ and $x + 2$ respectively at all other points, it is continuous everywhere else.

3. (12 points) Evaluate $Lim_{x \to \infty} \dfrac{x^2 + x + 1}{2x^2 - 3x + 15}$

   Solution: Divide above and below by the highest power, which is x^2. You get:

$$Lim_{x \to \infty} \frac{\frac{x^2+x+1}{x^2}}{\frac{2x^2-3x+15}{x^2}} = Lim_{x \to \infty} \frac{1 + \frac{1}{x} + \frac{1}{x^2}}{2 - \frac{3}{x} + \frac{15}{x^2}} = \frac{1+0+0}{2-0+0} = 1/2.$$

4. (Challenge) The Intermediate Value Theorem says that if the function $f(x)$ is continuous between $x = a$ and $x = b$ then it takes every value between $f(a)$ and $f(b)$. So for example, if $f(a)$ is negative and $f(b)$ is positive, then $f(x)$ will be zero (i.e, the graph wil cross the $x-$axis) for some x between a and b. Using this idea, show that $x^3 - 10x + 1$ equals zero (i.e, graph crosses the $x-$axis) three times between -4 and 4. [It is not enough just to draw a graph]

   Solution: We have $f(-4) = -23, f(-1) = 10, f(1) = -8, f(4) = 25$.

   So $f(x)$ goes from negative to positive, then back to negative and then again to positive.

   So it will become zero (graph will cross $x-$axis) between -4 and -1, then between -1 and 1 and again between 1 and 4.