4/3/09 Spring 09, Calculus II Test III Sitaraman

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

ANSWERS WITHOUT EXPLANATION WILL ONLY GET 40%

Time Limit 45 minutes

Please read the questions carefully before answering

It is recommended that you try those problems you are most comfortable with, first.

Attempt as many as you can; Anything over 100 is extra credit.

1. (10 pts) Find the general term of the following **sequence** starting with $n = 1$, determine if it converges and if it does find limit: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

2. (18 pts).State if true or false. Prove your answer if possible ; else provide counterexample:

a) If a_n goes to 0 as n goes to ∞, then Σa_n converges. (b) If $a_n = f(n)$ for $n \geq 1$ and $f(x)$ is a positive, continuous and decreasing function for $x \geq 1$, then $\sum_{k=1}^{\infty} a_k = \int_1^{\infty} f(x)\, dx$. (c) If the function $f(x) = \sum_{k=0}^{\infty} a_k x^k$, then $a_k = \frac{f^{(k)}(0)}{k!}$ in the interval where the series converges.

3.(20 pts) Find the sum of the following: (a) $\sum_{k=1}^{\infty} \frac{2}{3^k}$ (b) $\sum_{k=1}^{\infty} \frac{5}{2k(2k+2)}$

4. (20 pts) Check for convergence by any method: (a) $\sum_{k=1}^{\infty} \frac{k}{lnk}$ (b) $\sum_{k=1}^{\infty} \frac{e^2}{k^{\sqrt{2}}}$

5. (12 pts) (a) Check for convergence using comparison test: $\sum_{k=1}^{\infty} \frac{1}{k^2+2}$

(b) Check for convergence using ratio test: $\sum_{n=1}^{\infty} \frac{n!}{3^n}$

6. (20 pts) Find the Maclaurin series of $cosx$ (write first three terms and general term) using its derivatives. Find the radius of convergence.Use the Maclaurin series to evaluate the sum $1 - \frac{1}{2!} + \frac{1}{4!} - \frac{1}{6!} + \ldots$

7. [Challenge problem, 30 points] (a)Show: Only one of $\sum_{n=1}^{\infty} \frac{cos^2\theta}{n}$ or $\sum_{n=1}^{\infty} \frac{sin^2\theta}{n}$ can converge.(b) Now, it is known that $\sum_{n=1}^{\infty} \frac{cos2\theta}{n}$ converges. Using this, show that $\sum_{n=1}^{\infty} \frac{cos^2\theta}{n}$ diverges. [Don't use (b) to prove (a)].