2/27/08 Spring 09, Calculus II Test II solutions Sitaraman

1. (10 pts) Evaluate $\int \frac{sin(lnx)\ dx}{x}$

Soln: Let $u = lnx$. Then $du = dx/x$. So $\int \frac{sin(lnx)\ dx}{x} =$
$= \int sin(u)\ du = -cosu + C = -cos(lnx) + C$

2. (15 pts). Evaluate $\int x^2 lnx\ dx$ using integration by parts.

Soln: Let $u = lnx,\ dv = x^2 dx$. Then $du = dx/x,\ v = x^3/3$. Using integration by parts, we get $\int x^2 lnx\ dx = (lnx)(x^3/3) - \int (x^3/3)(1/x)dx$
$= (x^3 lnx)/3 - (1/3)\int x^2\ dx = (x^3 lnx)/3 - x^3/9 + C.$

3.(15 pts) Evaluate $\int sin^3 x cos^2 x\ dx$

Soln: Let $u = cosx$. Then $du = -sinxdx$ and $sin^2 x = 1 - u^2$. So given integral becomes $\int sin^2 x cos^2 x(sinxdx) = \int (1-u^2)u^2(-du)$
$= \int (u^4 - u^2)du = u^5/5 - u^3/3 + C = cos^5 x/5 - cos^3 x/3 + C.$

4. (15 pts) Evaluate $\int \frac{dx}{\sqrt{x^2+4x}}$ Use completing the square and trigonometric substitution.

Soln: First we complete the square for x^2+4x to get $(x+2)^2-2^2$. The given integral becomes $\int \frac{dx}{\sqrt{(x+2)^2-2^2}}$. Now we let $x+2 = 2sec\theta,\ dx = 2sec\theta tan\theta d\theta$, and $\sqrt{(x+2)^2 - 2^2} = \sqrt{4sec^2\theta - 4} = 2tan\theta$. The given integral then becomes $\int \frac{2sec\theta tan\theta d\theta}{2tan\theta} = \int sec\theta d\theta$. This equals $ln(sec\theta + tan\theta) + C = ln(\frac{x+2}{2} + \frac{\sqrt{x^2+4x}}{2}) + C.$

5. (15 pts) Evaluate $\int \frac{dx}{x^3-16x}$ using partial fractions.

Soln: $\int \frac{dx}{x^3-16x} = \int \frac{dx}{x(x^2-16)}$. Using partial fractions, $\frac{1}{x(x-4)(x+4)} = \frac{1}{32}\left(\frac{-2}{x} + \frac{1}{x-4} + \frac{1}{x+4}\right)$. So the required integral is the sum of the integrals of all these fractions and equals $(1/32)(-2ln|x| + ln|x-4| + ln|x+4|) + C = (1/32)ln\left|\frac{x^2-16}{x^2}\right| + C.$

6. (15 pts) Find the area under the curve $y = 1/\sqrt{x}$ from 0 to 1. Explain why this is an improper integral mathematically as well as using a graph.

Soln: This is improper because as x approaches 0, $1/\sqrt{x}$ approaches infinity. So the region of integration is an infinite region. Area is given by $\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+}\int_t^1 \frac{dx}{\sqrt{x}} = \lim_{t\to 0^+}[2\sqrt{x}]_t^1 = 2.$

7. (15 pts) Approximate $\int_1^6 \frac{e^x\ dx}{x}$ using the trapezoidal method, and 5 intervals.

The formula is $\frac{6-1}{10}[y_0+2y_1+2y_2+2y_3+2y_4+y_5]$ where $x_0 = 1, x_1 = 2, \ldots, x_4 = 5, x_5 = 6$ and $y_j = f(x_j) = \frac{e^{x_j}}{x_j}$. Plugging in the various values, we get the approximation $(1/2)[2.71828 + 7.38906 + 13.3904 + 27.2991 + 59.3653 + 67.2381322] = 88.70012$.

6. [Challenge problem, 20 points] Use a calculator to guess the value approached by $\frac{1}{n}^{1/n}$ as n goes to ∞. Then prove your answer by using calculus to find the limit.

Soln: This limit is same as that of $(1/x)^{1/x}$ as x approaches infinity. Letting $1/x = u$, this limit is same as that of u^u as u approaches zero. Now $ln(u^u) = uln(u)$ approaches zero as u approaches 0 [Need to use L'Hospital's rule here because lnu is undefined at $u = 0$!! Apply L'Hospital's to $ulnu = \frac{lnu}{1/u}$ and show that it goes to 0]. So u^u approaches $e^0 = 1$.