

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

ANSWERS WITHOUT EXPLANATION WILL ONLY GET 40%

Time Limit 45 minutes

Please read the questions carefully before answering

It is recommended that you try those problems you are most comfortable with, first.

Attempt as many as you can; Anything over 100 is extra credit.

1. (10 pts) Find the area enclosed between  $y = x$  and  $y = x^2$ .
2. (15 pts). Find the volume enclosed by the solid of revolution of the area enclosed between the curve  $y = \sqrt{\sin x}$  and the  $x$ -axis from 0 to  $\pi$ .
- 3.(15 pts) Find the length of the curve  $y = x^{3/2}$  from  $x = 0$  to  $x = 2$ .
4. (15 pts) Find the volume of the solid of revolution of the region between  $y = \cosh(x^2)$  and the  $x$ -axis from  $x = 0$  to  $x = 1$  about the  $y$ -axis using cylindrical shells.
5. (15 pts) Find the area of the surface obtained by revolving the curve  $y = \cos t, x = \sin t$  from  $t = 0$  to  $t = \pi/2$  about the  $x$ -axis.
6. (10 pts) Find the work done in stretching a spring of spring constant 20 a total length of 3 inches beyond its natural length. Your answer must be in pound-feet.
7. (20 pts) Integrate the following: (a)  $\int \frac{dx}{x^2-16}$  (b)  $\int \frac{x dx}{\sqrt{x^4+1}}$   
 [Hint for (b): Use  $\int \frac{dx}{\sqrt{x^2+1}} = \sinh^{-1} x$ .]
6. [Challenge problem, 20 points] Explain why the average of the values  $1, 1/2, 1/3, 1/4, 1/5, \dots, 1/n$  will always be bigger than  $\frac{\log(n+1)}{n}$  where  $n$  is any positive integer, using integration.