Spring 2010, Final Exam Howard University Mathematics
Math 156 Calculus I

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
PROVIDE STEP BY STEP SOLUTIONS TO GET FULL CREDIT
Time Limit 2 hours
Please read the questions carefully before answering

Section I
ANSWER ALL. EACH 10 POINTS

1. Find a formula for $f^{-1}(x)$ when $f(x) = 3x^3 - 5$
2. Explain why $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.
3. Is the function $\frac{x}{x^2+1}$ continuous everywhere? Explain your answer.
4. Find the equation of the tangent line to the curve $y = \frac{1}{x}$ at (1,1).
5. Find the limit using L'Hospital's Rule: $\lim_{x \to \infty} \frac{\ln(x)}{x^5}$
6. Find y' when $x^3 - 3xy + y^3 = 1$ using implicit differentiation.
7. Use any method to find the relative extrema of $\ln(1 + x^2)$.
8. Find the average value of $y = 1/x$ on the interval $[1, 2]$. [Average value of $f(x)$ over $[a, b]$ is $\frac{1}{b-a} \int_a^b f(x)dx$].

SECTION II
ANSWER ANY 6. EACH 20 POINTS

9. Compute the following limits:
   (a) $\lim_{x \to 3} \frac{x^2 - 6x + 9}{x - 3}$ (b) $\lim_{x \to \infty} \frac{x^2 - 6x + 9}{3x^2 - 5}$
10. Sketch a graph of the function $f(x)$ with the following features:
    (a) $\lim_{x \to -\infty} f(x) = 3$ (b) $\lim_{x \to -2} f(x) = \infty$ (c) $\lim_{x \to 1^-} f(x) = 2$
11. The population of a town after t years is given by $P(t) = \frac{1200}{1+e^{-0.1t}}$. Find the rate of change of this population after 5 years. Is it always increasing or decreasing? What happens to the population as t approaches ∞ ?

12. Find $\frac{dy}{dx}$ for the following functions.

a) $y = tan^{-1}(\ln(x))$ d) $y = e^{\sin^{-1}(x)}$

13. Find the differential of $f(x) = x^{\frac{1}{3}}$. Use your result to estimate $126^{\frac{1}{3}}$.

14. A man 5 ft. tall walks away from a street lamp that is 20 ft. above the ground. How fast is the tip of his shadow moving when he is 15 ft. from the lamp?

15. Sketch the graph of $y = x^2e^{-x}$. Label the critical points and points of inflection. State where it is increasing, decreasing, concave upward and concave downward. You must show all work to receive credit.

16. A closed rectangular container with a square base is to have a volume of 2250 cubic inches. The material for the top and the bottom of the container will cost \$ 2 per square inch. The material for the sides will cost \$ 3 per square inch. Find the dimensions of the container of least cost.

17. Show that there is no point c in the open interval (-1,8) such that $f'(c) = \frac{f(8)-f(-1)}{8-(-1)}$ where $f(x) = x^{2/3}$. Explain why this does not contradict the mean value theorem.

18. Evaluate the following integrals using the appropriate substitutions.

(a) $\int (2x-1)^3\,dx$ (b) $\int xe^{-x^2/2}\,dx$ (c) $\int \sin x \cos x\,dx$

19. Use the Fundamental Theorem of Calculus to evaluate the following integrals.

(a) $\int_0^1 (1+x^2)\,dx$ (b) $\int_0^1 \frac{1}{1+x^2}\,dx$ (c) $\int_0^1 \frac{x}{1+x^2}\,dx$