Spring 10, Calculus I Quiz 9 Howard University Mathematics
Math 156-05 Sitaraman 4/16/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 30 minutes
Please read the questions carefully before answering

NOTE: THIS QUIZ IS WORTH 40 POINTS (10 POINTS EXTRA CREDIT)

1. (10 points) Find the area under the graph of $f(x) = 3x^2$ from $x = 1$ to $x = 2$.

Soln:The area is $\int_1^2 3x^2\ dx = \left[x^3\right]_1^2 = 2^3 - 1^3 = 7.$

2. (10 points) Given that the velocity at time t of a particle is $v(t) = e^{-t}$ find the following: (a) distance and displacement from $t = 0$ to $t = 10$ seconds (b) acceleration at t seconds (c) displacement at time t seconds if it starts from $s(0) = 0$.

Soln: The displacement $\int_0^{10} e^{-t}dt = \left[-e^{-t}\right]_0^{10} = -e^{-10} - (-e^0) =$ $1 - 0.000045 = 0.999955$. The distance is equal to the displacement in this case because the velocity is always positive. The acceleration at t is the derivative of $v(t)$ which is $v'(t) = -e^{-t}$. The displacement at t is the integral of e^{-t} which gives $s(t) = -e^{-t} + C$. To get C put $t = 0$ in this. We get $s(0) = 0 = -e^0 + C$ from which we get $C = 1$. So $s(t) = 1 - e^{-t}$.

3.(10 points) The population of a town grows according to the formula $P(t) = 100e^{0.1t}$ after t years. Find the average value of the population in the first 5 years.

Soln: The average value is $\frac{1}{5}\int_0^5 100e^{0.1t}dt = \left[\frac{100e^{0.1t}}{0.1}\right]_0^5$
$= 200(e^{0.5} - 1) = 129.74$ or about 130 people.

4.(10 points) Evaluate $\int_0^5 \sqrt{3x+1}\ dx$ using substitution.

Soln: Let $u = 3x + 1$. Then $du = 3dx$ and $dx = du/3$. When $x = 0, u = 1$ and when $x = 5, u = \sqrt{3(5)+1} = 16$. So the given integral

is $\int_0^5 \sqrt{3x+1}\ dx = \int_1^{16} \sqrt{u}\frac{du}{3} = (1/3)\left[\frac{u^{(1/2)+1}}{(1/2)+1}\right]_1^{16} = (2/9)[16^{3/2} - 1^{3/2}] = (2/9)(63) = 14$