Spring 10, Calculus I Quiz 8 Howard University Mathematics
Math 156-05 Sitaraman 4/16/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 30 minutes
Please read the questions carefully before answering

NOTE: THIS QUIZ IS WORTH 40 POINTS (10 POINTS EXTRA CREDIT)

1. (10 points) Show that $f(x) = x^3$ satisfies the conditions for mean value theorem in [0,3]. Find a point $c \in (0,3)$ such that $f'(c) = \frac{f(3)-f(0)}{3-0}$.

Soln: $f'(c) = 3c^2$. If we want $f'(c) = \frac{f(3)-f(0)}{3-0}$ then we want $3c^2 = (27-0)/(3-0) = 9$, i.e, $c^2 = 3$. So the required $c = \sqrt{3}$. [Note that $-\sqrt{3}$ is not in (0,3)].

2. (10 points) Find the area under $f(x) = x^2$ from 0 to 1 (approximately) by dividing the region into 5 rectangles of length 0.2 each and heights $f(0.2), f(0.4), f(0.6), f(0.8), f(1)$ respectively. Compare this approximation with the exact area obtained using integration, namely $\int_0^1 x^2 dx$

Soln:Approximate area is $f(0.2)(0.2) + \ldots f(1)(0.2) = [0.2][(.2^2) + (.4^2)+(.6^2)+(.8^2)+1^2] = (0.2)[0.04+0.16+0.36+0.64+1] = 0.44$. Actual area is $\int_0^1 x^2 dx = [\frac{1^3}{3} - \frac{0^3}{3}] = 0.33$ where we are using $\int_0^1 f(x)dx = F(b) - F(a)$ with $F(x) = x^3/3$ because $F'(x) = 3x^2/3 = x^2$. [The accuracy will increase as we increase the number of intervals].

3.(10 points) Evaluate the following integrals by finding the antiderivatives: $(a) \int x^{1/2} dx \quad (b) \int e^{3x} dx.$

Soln: (a) $\int x^{1/2} dx = \frac{x^{(1/2)+1}}{(1/2)+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3} + C.$
$(b) \int e^{3x} dx = e^{3x}/3 + C.$
You can check both answers by differentiating them.

4.(10 points) Integrate using substitution: (a) $\int \frac{dx}{x-1}$ (b) $\int \sin x \cos x dx$.

Soln:(a) To find $\int \frac{dx}{x-1}$ let $x - 1 = u$ then $dx = du$ and $\int \frac{dx}{x-1} = \int \frac{du}{u} = ln|u| + C = ln|x-1| + C$.

(b) To find $\int \sin x \cos x dx$ let $\sin x = u$. Then $\cos x dx = du$ and $\int \sin x \ \cos x dx = \int u du = u^2/2 + C = (\sin x)^2/2 + C$