Spring 10, Calculus I Quiz 4 Howard University Mathematics
Math 156-05 Sitaraman 2/26/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 5 points unless otherwise stated.

1. Using the fact that $\underset{x\to 0}{Lim}\frac{sin x}{x} = 1$ show that $\underset{x\to 0}{Lim}\frac{sin 3x}{x} = 3$.

Soln: Letting $u = 3x$ and noting that $u \to 0$ as $x \to 0$, we get $\underset{x\to 0}{Lim}\frac{sin 3x}{x} = \underset{u\to 0}{Lim}\frac{sin u}{u/3} = 3\underset{u\to 0}{Lim}\frac{sin u}{u} = 3(1) = 3$.

2. (10 points) For the function $y = f(x) = x^3$ find the following: (a) average rate of change from $x = 1$ to $x = 1.1$. (b) average rate of change from $x = 1$ to $x = 1.01$. (c) Instantaneous rate of change at $x = 1$ using the limit formula for derivative.

Soln: (a) Average rate of change $= \frac{f(1.1)-f(1)}{1.1-1} = \frac{1.1^3-1}{.1} = 3.31$.

(b) Average rate of change $= \frac{f(1.01)-f(1)}{1.01-1} = \frac{1.01^3-1}{.1} = 3.03$.

(c) Using limit formula the instantaneous rate of change is $f'(x) = \underset{h\to 0}{Lim}\frac{f(x+h)-f(x)}{h} = \underset{h\to 0}{Lim}\frac{(x+h)^3-x^3}{h} = \underset{h\to 0}{Lim}\ 3x^2 + 3xh + h^2 = 3x^2$. Setting $x = 1$ we get $f'(1) = 3(1)^2 = 3$.

3.(10 points) Find the equation of the tangent line to the graph of $y = x^2 + 2x$ at $x = 1$ using the derivative $f'(x)$ at $x = 1$. [also called $f'(1)$]. You don't need to use limit formula.

Soln: $f'(x) = (x^2)' + (2x)' = 2x + 2$ and so slope at $x = 1$ is $f'(1) = 2(1) + 2 = 4$. When $x = 1$ we have $y = (1)^2 + 2(1) = 3$. So the equation of tangent line at $x = 1$ is $(y - 3) = 4(x - 1)$ which after simplifying gives $y = 4x - 1$.

4. If the function $f(x)$ has a horizontal tangent at $x = a$, find the instantaneous rate of change of f at $x = a$. [i.e, find $f'(a)$.] Give an example of a function with a horizontal tangent at a point.

Soln: If tangent is horizontal then slope of tangent is 0. But slope of tangent is same as instantaneous rate of change which is same as derivative. So $f'(a) = 0$. An example is $f(x) = x^2$, the parabola. At

$x = 0$ the parabola's tangent is the $x-$axis. In fact, $f'(0) = 2(0) = 0$ because $f'(x) = 2x$.