Spring 10, Calculus I Quiz 2 Howard University Mathematics
Math 156-05 Sitaraman 1/29/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 5 points unless otherwise stated.

1. Graph $y = -3sin(x + \frac{\pi}{2})$ showing amplitude, period and phase shift.

Soln: If $y = Asin(Bx+C)$, amplitude is $|A|$. In this case amplitude is 3. Period is $2\pi/B = 2\pi/1 = 2\pi$. Phase shift is $-C/B = -\frac{\pi}{2}$. First graph $3sin(x+\frac{\pi}{2})$ using the amplitude, period and phase shift as above. Phase shift $-\frac{\pi}{2}$ means the sine wave will start at $-\frac{\pi}{2}$. Once you have the graph of $3sin(x+\frac{\pi}{2})$ reflect it (flip it) about the $x-$axis to get the graph of $-3sin(x+\frac{\pi}{2})$.

2. Find the inverse $f^{-1}(x)$ of $f(x) = 1 - x^2, x \geq 0$ Find its domain and range and $f^{-1}(1)$.

Soln: Note that the graph is the right half of a parabola facing down with vertex at (0,1). $y = 1 - x^2$ means $x^2 = 1 - y$ and $x = \sqrt{1-y}$. We take the positive square root because $x \geq 0$. In other words, range of $f^{-1}(x) = \sqrt{1-x}$ equals the domain of f which is all real x such that $x \geq 0$. The domain of f^{-1} is x such that $1 - x \geq 0$ or $x \leq 1$. This can also be found by saying range of f is all possible $y-$values for $y = 1 - x^2$ which is $[1, -\infty]$.

3. Given that $\theta = cos^{-1}(-0.5)$, find $sin\theta$ and $tan\theta$.

Soln: We get, by definition, $cos\theta = -0.5$. Since the range of cos^{-1} is $[0, \pi]$, this means θ is in the second quadrant. Then if α is the reference angle [obtained by subtracting θ in degrees from 180] we have $-cos\alpha = cos\theta = -1/2$. So $cos\alpha = 1/2$ and so from a triangle with adjacent side 1 and hypotenuse 2 we get opposite side $\sqrt{3}$ and hence $sin\alpha = \sqrt{3}/2$. Since θ is in second quadrant, $sin\theta = sin\alpha = \sqrt{3}/2$. Then $tan\theta = sin\theta/cos\theta = -\sqrt{3}$. You can also find $sin\theta$ by solving $sin^2\theta + cos^2\theta = 1$ and then choosing the appropriate sign.

4. Write as a single logarithm and evaluate:

$ln(x+7) + ln(x-7) - ln(x^2-49)$.

Soln: Using properties of logarithm, we get $ln(x+7) + ln(x-7) - ln(x^2-49) = ln[(x+7)(x-7)/(x^2-49] = ln[\frac{x^2-49}{x^2-49}] = ln(1) = 0$.

5. (10 points) The amount of \$1000 deposited in a certain fund grows according to the formula $A(t) = 1000(1.1)^t$. Find the amount after 3 years. When will it equal 2000?

Soln: $A(3) = 1000(1.1)^3 = 1331$ dollars.To find when it becomes 2000, we set $A(t) = 2000$ and solve for t. We get $2000 = 1000(1.1)^t$ which simplifies to $2 = 1.1^t$. Now taking logarithms on both sides, we get $ln2 = tln[1.1]$ from which we get $t = ln2/ln[1.1] = 7.27$ years or approximately 7 years 3 months 8 days.