Spring 10, Calculus Quiz 1 Howard University Mathematics
Math 156-05 Sitaraman

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 30 minutes
Please read the questions carefully before answering
Each problem 5 points unless otherwise stated.

1. Find the composition $f \circ g$ of the functions $f(x) = x^2 - 1$ and $g(x) = \sqrt{x}$. Find the domain of $f \circ g$.

Soln: $f \circ g(x) = f(g(x)) = (g(x))^2 - 1 = ((\sqrt{x})^2 - 1) = x - 1$. To find the domain, first x must be in the domain of g which means $x \geq 0$. Then $g(x)$ must be in the domain of f. But since domain of f is all real numbers, the domain of $f \circ g$ is simply the same as the domain of g which is all non-negative real numbers, i.e, $x \geq 0$.

2. Express the height of a point on a semi-circular arch of radius 10 meters as a function of the angle between the line joining the point to the centre and the ground. [See figure below]. What is the height if the angle is 30°?

Soln: From the triangle in the picture, we have $sin\theta = h/10$. So $h = 10sin\theta$. If $\theta = 30$, then $h = 10(sin30°) = 10(1/2) = 5$ meters. Note that the hypotenuse in the triangle is also a radius. So the hypotenuse is 10m.

3.(10 points) Given that $f(x) = \begin{cases} 2x - 1 & x \leq 1 \\ x^2 & x > 1 \end{cases}$, graph $f(x)$.

Also find $f(0), f(2.3), f(1)$.

Soln: This is a piecewise defined function. The graph is a straight line given by $y = 2x-1$ upto $x = 1$ with slope 2 and $y-$intercept -1 and x-intercept $1/2$. It is a parabola from $x = 1$ onwards. $f(0) = 2(0)-1 = -1$ because $0 \leq 1$. For same reason $f(1) = 2(1)-1 = 1$. But because $2.3 > 1$, we have to use the $y = x^2$ part and get $f(2.3) = 2.3^2 = 529$.

4. (10 points) Starting with the graph of $y = 2x$, graph $y = 1-2x$. Show all the intermediate graphs.

Soln: To get the graph of $y = 1 - 2x$ flip the graph of $y = 2x$ about the $y-$axis and then move it up by 1. The resulting graph should pass through (0,1), (1, -1).