3. Evaluate the following integrals:

(a) $\int cosh^3 x sinhx\ dx$ (b) $\int \frac{dx}{\sqrt{x^2-1}}$

Soln: (a) Using the substitution $u = coshx$, we get $sinhx\ dx = du$ and hence $\int cosh^3 x sinhx\ dx = \int u^3 du = u^4/4 = \frac{cosh^4 x}{4} + C$. (b) It was shown in class that this integral is equal to $cosh^{-1}x + C$.

2. Evaluate $\int \frac{e^x dx}{\sqrt{e^{2x}-4}}$ using a suitable substitution and applying the relevant integration formula.

Let $u = e^x/2$. Then $du = (e^x/2)dx$. Also $\sqrt{u^2-1} =$
$\sqrt{(e^x/2)^2-1} = \sqrt{\frac{e^{2x}}{4}-1} = \frac{\sqrt{e^{2x}-4}}{2}$.

So the given integral becomes

$$\int \frac{e^x dx}{\sqrt{e^{2x}-4}} = \int \frac{2du}{2\sqrt{u^2-1}} = \int \frac{du}{\sqrt{u^2-1}} = cosh^{-1}u = cosh^{-1}(e^x/2).$$

CHECK YOUR ANSWER BY DIFFERENTIATING!!

3. Evaluate $\int x^2 sinx\ dx$ using integration by parts twice.

First let $v = x^2, du = sinx\ dx$. Then $dv = 2xdx, u = \int sinx\ dx = -cosx$. We have

$$\int vdu = uv - \int udv$$

$$= x^2(-cosx) - \int (-cosx)(2xdx) = -x^2 cosx + 2\int xcosx\ dx.$$

Now integrate by parts again to get $\int xcosx\ dx$. Let $v = x, du = cosx\ dx$. Then $dv = dx, u = \int cosx\ dx = sinx$. We get $\int xcosx\ dx = xsinx - \int sinx\ dx = xsinx + cosx$. Plugging in this answer for $\int xcosx\ dx$ into the expression above we get

$$\int x^2 sinx\ dx = -x^2 cosx + 2\int xcosx\ dx = -x^2 cosx + 2xsinx + 2cosx + C.$$