

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

Time Limit 45 minutes

Please read the questions carefully before answering

Each problem 10 points unless otherwise stated.

Total 120 points including 20 points extra credit for challenge problem.

1. (20 points) A rectangular field has a fence of total length 202 meters. [In other words, its perimeter is 202]. If the area is 2109.25 square meters, find the length and width.

Soln: Let l equal the length and w the width. Then perimeter is $2(l+w) = 202$ which means $l+w = 101$ and so $l = 101 - w$. Then area is $2109.25 = w(101 - w) = 101w - w^2$ which gives $w^2 - 101w + 2109.25 = 0$. Using quadratic formula and taking the positive value we find that $w = 29.5$ and then $l = 101 - w = 71.5$ meters.

2. Write the equation of line passing through $(1.25, -2.1)$ and $(3.25, 1.9)$. You must first find the slope.

Soln: The slope is $\frac{y_1 - y_2}{x_1 - x_2} = \frac{-2.1 - 1.9}{1.25 - 3.25} = -4 / -2 = 2$. The equation is of the form $y = 2x + b$. Plugging in $(1.25, -2.1)$ we get $-2.1 = 2(1.25) + b$ and solving for b we get $b = -2.1 - 2.5 = -4.6$. So the equation is $y = 2x - 4.6$.

3. Find the standard form equation of circle with center $(1,1)$ and passing through $(2,2)$. [Hint: Radius is the distance between these two points].

Soln: Radius using distance formula is $r = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$. So the equation in standard form is $(x-1)^2 + (y-1)^2 = (\sqrt{2})^2 = 2$.

4.(20 points) Find the center and radius and graph the circle whose equation is $x^2 + y^2 - 4x - 6y = 12$.

Soln: First we gather the x and y terms together to get $(x^2 - 4x) + (y^2 - 6y) = 12$. Completing the square, we get $(x-2)^2 - 2^2 + (y-3)^2 - 3^2 = 12$. Simplifying, we get the standard form equation as $(x-2)^2 + (y-3)^2 = 25$. This tells us that the center is $(h, k) = (2, 3)$ and the radius is $\sqrt{25} = 5$.

5. The time required to process a shipment varies directly with the weight of the load and inversely with the number of workers assigned. If 3000 lbs can be processed by 6 workers in 8 hrs, how long would it take for 5 workers to process 4000 lbs?

Soln: Let T be the time taken, n the number of workers, and w the weight. Then $T = kw/n$ where k is the constant of variation. We have at first $T = 8, w = 3000, n = 6$. Plugging in, we get $8 = k(3000)/6$. So $k = 8/500 = 0.016$ and thus the time for 4000 lbs with 5 workers is $T = 0.016(4000)/5 = 12.8$ hours.

6. Looking at the graph of the function f below find the following: (a) the domain of f (b) The x and y intercepts (c) Where is function increasing or decreasing (d) symmetry about x, y axes and the origin.

Soln:(a) Domain is $[-2,2]$ (b) Both x and y intercepts are 0. (c) Function is decreasing from -2 to 0 and increasing from 0 to 2. (d) It is symmetric about y -axis.

7. Given that $f(x) = x + 5$ and $g(x) = x^2$ find $(f + g)(x), (f - g)(0), (fg)(1), f(x - 5)$.

Soln: We have $(f+g)(x) = (x+5)+x^2 = x^2+x+5$. Also $(f-g)(x) = x + 5 - x^2$ and $(f - g)(0) = 5$. Now $(fg)(x) = (x + 5)x^2 = x^3 + 5x^2$. So $(fg)(1) = 1^3 + 5(1^2) = 1 + 5 = 6$. Finally $f(x - 5) = (x - 5) + 5 = x$.

8. The cost of renting a car is given by $C(x) = 0.35x + 50$ where x is the number of miles driven. (a) Find the cost if 100 miles are driven. (b) How many miles were driven if the total cost was 200 dollars?

Soln:(a) $C(100) = 0.35(100) + 50 = 85$. (b) If $C(x) = 200$, then $200 = 0.35x + 50$ which means $150 = 0.35x$ which gives $x = 150/0.35 = 428.57$ miles.