Spring 10, College Algebra I Test 2 Howard University Mathematics
Math 006-19 Sitaraman 3/5/2010

Instructions:
PLEASE PROVIDE STEP BY STEP EXPLANATIONS
Time Limit 45 minutes
Please read the questions carefully before answering
Each problem 10 points unless otherwise stated.
Total 120 points including 20 points extra credit for challenge problem.

1. Rationalize the denominator of $\frac{3}{4\sqrt{5}-1}$.

Soln: Multiplying above and below by the "conjugate" $4\sqrt{5}+1$ we get $\frac{3(4\sqrt{5}+1)}{(4\sqrt{5}-1)(4\sqrt{5}+1)} = \frac{12\sqrt{5}+3}{(4\sqrt{5})^2-1^2} = (12/79)\sqrt{5}+(3/79)$.

2. Solve for x : $2x-15 = (1-x)/2$.

Soln:Multiplying both sides by 2 we get $4x-30 = 1-x$. When solving equations, you want to bring x to one side. So we bring the $-x$ to the left and the -30 to the right. We get $5x = 31$ and thus $x = 31/5 = 6.2$.

3. Solve for x : $(x-3)^2 = 16$.

Soln: Taking square root of both sides, we get $x-3 = \pm\sqrt{16}$. So $x = 3+4 = 7$ or $x = 3-4 = -1$.

4.(20 points) Solve $x^2+4x+3 = 0$ in two ways: First by factoring, then by completing the square.

Soln:(a)By factoring: $x^2+4x+3 = (x+3)(x+1) = 0$ means $x+3 = 0$ or $x+1 = 0$. This means $x = -3$ or $x = -1$. (b) By completing square: $x^2+4x = -3$. Adding the square of half the $x-$coefficient to both sides, we get $x^2+4x+4 = -3+4$ which gives $(x+2)^2 = 1$ which means $x+2 = 1$ or $x+2 = -1$ upon taking square root of both sides. Thus $x = -3$ or $x = -1$.

5.(20 points) A person has a total of 5000 dollars to invest. He invests some in stocks and the rest in bonds. If stocks give 8 percent return and bonds give 4 percent return and he has 300 dollars at the end of the year, how much did he invest in each?

Soln: Let x be the amount in bonds. Then $5000 - x$ is the amount in stocks. Total return is return from stocks plus return from bonds. So we have $300 = x(0.04) + (5000 - x)(0.08) = 400 - 0.04x$. Simplifying and bring x to one side, we get $0.04x = 100$ and thus $x = 100/0.04 = 2500$.

6. Solve by making it look like a quadratic: $x^4 + 3x^2 + 1 = 0$.

Soln:Let $u = x^2$. Then $u^2 = (x^2)^2 = x^4$. Thus the given equation becomes $u^2 + 3u + 1 = 0$. Then we can solve it using quadratic formula with $a = 1, b = 3, c = 1$. We get $x = x = \frac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)} = \frac{-3 \pm \sqrt{5}}{2}$ and the two solutions are $\frac{-3+\sqrt{5}}{2}$ and $\frac{-3-\sqrt{5}}{2}$. Both of these values for u are negative. Since $u = x^2$ that means we can find solutions for x^2 but not x because for any x its square will not be positive. So here are no solutions for x.

7. Solve for $x : |x - 2| < 1$. Write the answer in interval notation.

Soln: We have $-1 < x - 2 < 1$. So $-1 + 2 < x < 1 + 2$ or $1 < x < 3$. In interval notation the solution is (1,3).

8. The length of a rectangular room is 5 feet more than the width. The area is 100 square feet. Find the length and the width.

Soln:Let w be the width. Then the length is $w + 5$ and the area is $w(w + 5) = 100$. This gives $w^2 + 5w = 100$ which gives $w^2 + 5w - 100 = 0$. Solving using quadratic formula we get $w = \frac{-5 \pm \sqrt{5^2 - 4(1)(-100)}}{2(1)} = \frac{-5 \pm \sqrt{425}}{2}$ and the two solutions are $\frac{-5+\sqrt{425}}{2}$ and $\frac{-5-\sqrt{425}}{2}$. Length has to be positive so we take the positive one which equals 7.80776 and then the width which is w+5 equals 13.80776.

9. [Challenge problem, 20 points extra credit] Find the value of the number $\sqrt{2 + \sqrt{2 + \sqrt{2 + \ldots}}}$. Hint: Call this number x and then come up with a quadratic equation. Note that the square rooting process goes on infinitely.

Soln: Since it goes on infinitely, x actually satisfies the equation $\sqrt{2 + x} = x$. Squaring both sides, we get $x + 2 = x^2$. Solving using quadratic formula, we get $x^2 - x - 2 = 0$ means

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)} = \frac{1 \pm \sqrt{9}}{2}$ and again taking the positive solution we get $x = 2$. This equation can also be solved by factoring.