Spring 10, College Algebra I Test 1 Solutions
Math 006-19 Sitaraman 2/12/2010

1. Simplify the following and write answer with positive exponents:
(a) $\frac{2^{65}2^{59}}{2^{121}}$ (b) $(xyz)^{30}y^{-20}z^{43}$

Soln: (a) $\frac{2^{65}2^{59}}{2^{121}} = 2^{65+59-121} = 2^{-3} = \frac{1}{8}$.
(b) $(xyz)^{30}y^{-20}z^{43} = x^{30}y^{30}z^{30}y^{-20}z^{43}$
$= x^{30}y^{30-20}z^{30+43} = x^{30}y^{10}z^{73}$.

2. (a) Write 10345 in scientific notation (b) Write 2.4×10^{-4} in decimals.

Soln:(a) 1.0345×10^4 because you have to move the decimal 4 places.
(b) 0.00024

3. The hypotenuse and one side of a right angled triangle are 13 and 5 respectively. Find the 3rd side and the area.

Soln: $c^2 = a^2 + b^2$ with $c = 13, a = 5, b$ unknown. So $b^2 = 13^2 - 5^2 = 144 = 12^2$. So $b = 12$. Area is 1/2 times base times height and in this case it is $1/2(12)(5) = 30$ units.

4.(20 points) Simplify after addition or subtraction:
(a) $(x^3 - 1) - (x^2 - 2x + 5)$ (b)$\frac{1}{x-3} - \frac{x}{3-x}$

Soln:(a)$x^3 - 1 - x^2 - (-2x) - 5 = x^3 - x^2 + 2x - 6$ (b) $\frac{1}{x-3} - \frac{x}{3-x} = \frac{1}{x-3} + \frac{x}{x-3}$ because $3 - x = -(x - 3)$. Since the denominators are the same we can add the numerators and get $\frac{x+1}{x-3}$.

5. Factor completely $6x^2 - 7x - 5$.

Soln: We have 6(-5) = -30. So we try to find two numbers A,B so that AB = -30 and A+B = -7. We find that 3 and -10 work. So we write the middle term as a sum to get $6x^2 - 7x - 5 = 6x^2 + 3x - 10x - 5$. We group the two terms at a time and then factor out common terms: $(6x^2 + 3x) + (-10x - 5) = 3x(2x + 1) - 5(2x + 1) = (3x - 5)(2x + 1)$. Check that answer is right by multiplying.

6. Expand $(7xy - 2z)^2$ using the formula $(F - S)^2 = F^2 - 2FS + S^2$.

Soln:$(7xy)^2 - 2(7xy)(2z) + (2z)^2 = 49x^2y^2 - 28xyz + 4z^2$.

7. (20 points) Add and simplify as much as possible : $\frac{x^2}{x^2-4} - \frac{x+1}{x+2}$.

Soln:$\frac{x^2}{x^2-4} - \frac{x+1}{x+2} = \frac{x^2}{x^2-2^2} - \frac{x+1}{x+2}$

$= \frac{x^2}{(x-2)(x+2)} - \frac{x+1}{x+2} = \frac{x^2}{(x-2)(x+2)} - \frac{(x+1)(x-2)}{(x+2)(x-2)} = \frac{x^2-(x^2-x-2)}{(x-2)(x+2)} = \frac{x+2}{(x-2)(x+2)} = \frac{1}{x+2}$.

8. Simplify: $\left(3x^{1/3}y^{4/3}z^{2/3}\right)^3$.

Soln: $\left(3x^{1/3}y^{4/3}z^{2/3}\right)^3 = 3^3(x^{1/3})^3(y^{4/3})^3(z^{2/3})^3 = 27xy^4z^2$. Note that we multiply the exponents by 3 because when you raise a power to another power you multiply the exponents.

9. [Challenge problem, 20 points extra credit] Show that if a polynomial $x^3 + ax^2 + bx + c$ is divisible by $x - m$ where m is any integer, then $m^3 + am^2 + bm + c = 0$ by using synthetic division. [In fact, this is true for any polynomial of any degree: if $(x - m)$ divides $p(x)$, then the number you get by plugging in m [also called $p(m)$] is zero].

Soln:

m	$\vert$	1	a	b	c
	$\vert$	0	m	$am + m^2$	$bm + am^2 + m^3$
		1	$m + a$	$b + am + m^2$	$c + bm + am^2 + m^3$

The remainder is $c+bm+am^2+m^3$. If $(x-m)$ divides x^3+ax^2+bx+c then remainder should be zero. Hence $c + bm + am^2 + m^3 = 0$.