

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

Time Limit 30 minutes

Please read the questions carefully before answering

Each problem 5 points unless otherwise stated.

1. (10 points) John and Kim can finish a job in 5 hours working together. Kim alone takes 7 hours. How long will it take for John to do it by himself?

Soln: Let x be the time taken by John by himself. Then $1/x$ is the amount of work done by John in one hour. Similarly $1/7$ is the amount of work done by Kim in one hour and $1/5$ is the amount they do together in one hour. Since amount of work done together in one hour = work done by John in one hour + work done by Kim in one hour, we get $\frac{1}{5} = \frac{1}{x} + \frac{1}{7}$. Now we just need to solve for x . We first get x by itself on one side: $\frac{1}{x} = \frac{1}{5} - \frac{1}{7}$ which means $\frac{1}{x} = \frac{2}{35}$. So taking reciprocals we get $x = 35/2 = 17.5$ So John takes 17 1/2 hours to do the job by himself.

2. (8 points) A ship going against the wind takes 5 hours to get to a port. On the return trip with the wind pushing it forward it takes 3 hours for the same distance. If the ship has a constant speed of 20 mph, and the speed of the wind is w , set up an equation to find w . You don't need to find the value of w .

Soln: We use distance = speed times time. Actual speed on the trip to the port is $20 - w$ because it is going against the wind. So distance is $(20 - w) \times 5$. Actual speed on the return is $(20 + w)$ and so the distance is $(20 + w) \times 3$. Since they go the same distance we get $(20 - w) \times 5 = (20 + w) \times 3$. Simplifying, we get $100 - 5w = 60 + 3w$.

3.(6 points) Find the distance between the points (1,-6) and (13, -1) using the distance formula. Also find their mid-point.

Soln: $d = \sqrt{(1 - 13)^2 + (-6 - -1)^2} = \sqrt{(-12)^2 + (-5)^2} = \sqrt{169} = 13$ units. The midpoint is $(\frac{1+13}{2}, \frac{-6-1}{2}) = (7, -3.5)$.

4. (6 points) Test the curve represented by $y - x^2 + x = 0$ for symmetry about the x -axis, y -axis, and the origin. Do not use graph (you can check answer by graphing if you wish).

Soln: Symmetry about x -axis: Change y to $-y$: $(-y) - x^2 + x = 0$ changes the equation. So it is not symmetric about y -axis.

Symmetry about y -axis: Change x to $-x$: $y - (-x)^2 + (-x) = y - x^2 - x = 0$. This also changes the equation. So no symmetry about y -axis either.

Symmetry about $(0,0)$: Change both: $(-y) - (-x)^2 - x = 0$ which upon multiplying by -1 gives $y + x^2 + x = 0$. This also changes the equation. So no symmetry about the origin either.

5. (Bonus 10 points) Find the x and y intercepts of $y - x^2 + x = 0$ without using graph. (You can check answer by graphing if you wish).

Soln: For x -intercept put $y = 0$. You get $0 - x^2 + x = 0$ from which you get $-x^2 + x = 0$. Solve for x by factoring: $x(-1+x) = 0$ means $x = 0$ or $-1 + x = 0$ which means $0, 1$ are the x -intercepts. For y -intercept put $x = 0$. You get $y - 0 = 0$ or $y = 0$. So 0 is the only y -intercept.